EFFECTIVE School Practices

VOLUME 13, NUMBER 2 PRICE $5.00 SPRING 1994

FOCUS: ACHIEVING HIGHER STANDARDS IN MATHEMATICS

FORMERLY ADI NEWS

Philosophy of *Effective School Practices*

1. Teachers are responsible for student learning.
2. The curriculum is a critical variable for instructional effectiveness.
3. Effective teaching practices are identified by instructional research that compares the results of a new practice with the results of a viable alternative.
4. Experiments should not be conducted using an entire generation of Americans. The initial experimentation with a new practice should be small in scale and carefully controlled so that negative outcomes are minimized.
5. A powerful technology for teaching exists that is not being utilized in most American schools.

Effective School Practices (formerly *ADI News*) is a publication of the Association for Direct Instruction. The mission of the Association for Direct Instruction, as stated in the by-laws, is to promote the improvement of educational methods.

The name *Direct Instruction* originated with the highly effective instructional model first developed by Zig Engelmann in Project Follow Through during President Johnson's Great Society legislation. Although the evaluation of Project Follow Through showed the Direct Instruction model to be far more effective than the other models on every identified outcome, education in America remained generally unchanged.

A few educators, impressed by the extraordinary results of the original Direct Instruction model and the programs that were developed as DI evolved, formed the Association for Direct Instruction in 1981.

Today, this organization is a vanguard in promoting school practices that have been validated as effective through the use of the scientific method in educational research.

The Association for Direct Instruction was incorporated in 1981 in the state of Oregon for educational purposes. ADI is a non-profit, tax-exempt corporation under Section 501(c)3 of the Internal Revenue Code and is a publicly supported organization as defined in Sections 170(b)(1)(A)(ii) and 509(a)(1). Donations are tax-deductible.

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From the Field: Letters

To the editor:

There was an error in the article entitled "Do Word Processors Really Make Good Spellers Obsolete?" in the Winter 1994 issue. The sentence in error stated that spelling aides seemed "particularly inadequate when the student is a poor speller with a weak vocabulary." My intention was to say that "the aides seem particularly inadequate when the student is a poor speller with a *developed* oral vocabulary." My students generally have more words in their oral vocabulary than they can spell. For example, the first-graders in my resource room have attempted to spell words such as *ancient* and *hallucination*. Word processors and electronic spelling aides are not very helpful for these students.

Berta Bender
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To the editor:

I would like to print, in their entirety, several articles from your excellent publication, "Effective School Practices." Specifically, I wish to use:

"Overview: Toward World Class Standards" from the Summer 1993 issue,

"A Whole Lotta Learning Going On" from the same issue,

"Child-Directed Teaching Methods: A Discriminatory Practice of Western Education" from the Spring 1993 issue.

I am compiling a research and information handbook for the members of the National Parents Commission to use in debating education issues with education bureaucrats. Your articles offer a clear picture of what validated research is, what instructional child-directed methodologies are, with their repercussions, and an example of a program proven to be successful.

Thank you for your excellent work in education.

Peg Luksik, Chairman
National Parents Commission
Home Office
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To the editor:

I have had the opportunity to review a new mathematics program that I think has many of the features that Bob Dixon identified as important things to look for in adopting an effective mathematics curriculum [See Bob Dixon's article, this issue]. The program is by Charles Ledger, is published by Irwin Publishers in Toronto, and is called the *Spirit of Math*. The author, Charles Ledger, is a teacher who developed the program over his years of teaching. His students have won many awards for their performance in mathematics contests in Toronto. The first level will be out this fall for seventh grade. Other levels will follow. I believe it will be a program for middle school grades 7 through 9 when it is finished.

Andrew Nikiforuk
Education Columnist
Canadian Globe and Mail

Focus: Achieving Higher Standards in Mathematics

Overview
Bernadette Kelly, Guest Co-Editor
University of Oregon

The last issue of Effective School Practices had a double focus: *Working With What Works*, and *Beginning Reading Instruction*. The first section of the issue detailed how current reform movements seem to be focused on implementing the latest fads rather than research-based practices. The second section articulated these same issues as they apply to the specific content area of beginning reading. The focus of this current issue is effective *mathematics* instruction. It includes a variety of articles that articulate both problems with current popular approaches and possible solutions.

A fundamental tenet of Direct Instruction is that we learn through a combination of positive and negative examples. This combination gives rise to a clearer definition of a concept. It is hoped that the combination of papers provided in this issue will provide the reader with a clear perspective on the issues surrounding the math reform movement debate, as we seek to provide examples of what effective mathematics instruction is, and is not.

In his article, *What if Those Math Standards are Wrong?* (p. 5), Chester E. Finn, Jr., expresses concern arising from the "lemming-like" acceptance of the National Council of Teachers of Mathematics (NCTM) Standards. Perhaps Finn's most serious concern is the lack of a research base for the NCTM recommendations. Without a solid research base demonstrating that the NCTM recommendations are effective and will accomplish the desired results—"more numerate young Americans"—we run the risk of implementing new standards that will not serve either teachers or students well.

Another concern relates to the quality of the Standards themselves. The aspects of math emphasized and de-emphasized by the NCTM might result, he fears, in children who "might have all sorts of imaginative ideas about tackling a problem, yet seldom get the right answer to it because 5 times 11 was beyond their ken." His concern echoes that of Richard Martin two decades ago when critiquing the new math movement: "There is one slight hitch: many of these kids can't add, subtract, multiply or divide." Finn questions the lurching from one extreme to another in education and calls for a balance: "Great teachers ... attend both to problem-solving strategies *and* rapid calculation of the right answer in math class. They know these fierce arguments involve spurious choices and phony tradeoffs."

The NCTM describe their unvalidated recommendations for teaching practice as a "vision." The vision articulated by the NCTM is being implemented nationwide. How well does the vision translate into "new" practice? Apparently, not very well.

Jay Mathews' article, *Divide and Confound*, (p. 8), is written from the perspective of a concerned parent. He sees the results of the mathematics instruction his daughter, Kate, has received, and is no less than alarmed. The math program used at Kate's school is based on the Standards. Part of the NCTM vision is to help students "trust in their own mathematical thinking." The math program Kate is working in has apparently accomplished that—when asked to add 8 plus 5, "Kate put on an 8-year old's favorite armor; a look of utter disbelief at the ignorance and misplaced values of adults. Are you kidding? she said. It's 12."

While *Divide and Confound* is written from the perspective of a concerned parent, *Déjà Vu?—The New "New Math"* (p. 11), is written from the perspective of a concerned educator and expresses the same concern—that in math education we seem to suffer from specific amnesia—an inability to learn from our mistakes. As Mathews says, "No idea, no matter how battered, ever really dies in American education."

The *Déjà Vu* paper explores the difficulties experienced when teachers attempt to translate the California *Math Framework* into practice. The paper critiques a series of four articles written by educational researchers who have a strong commitment to the vision and philosophy of the NCTM. The same vision and philosophy is articulated in the California *Framework*.

There seems to be a serious dissociation between the rhetoric of the *Framework* and actual student-performance goals. There is a danger that without clearly articulated goals for student performance and methods of evaluating whether those goals have been met, there is room for a loss of accountability on the part of teachers and instructional programs. This is reflected in the fact that the authors, when observing the teachers in the classroom, failed to attend to student learning: "Because we did not collect student data, we cannot tell what students learned or did not learn." (Putnam, Heaton, Prawat, & Remillard, 1992, p. 217). What they did observe, however, were teachers who, when attempting to apply the rhetoric of the *Framework*, lacked the sophistication to deal with the instructional situations they themselves devised. In attempting to create challenging and creative problem-solving activities, two teachers unintentionally facilitated the formation of misconceptions for perimeter, volume and average.

Meeting the NCTM Standards through Direct Instruction: Connecting Math Concepts (p. 20) illustrates how the design of the *Connecting Math Concepts* (CMC) program can provide teachers with a reliable tool to help accomplish the goals of the NCTM. By introducing important content in a logical, sequential, strategic fashion, students can experience success in the areas of increased emphasis outlined by the NCTM, such as problem solving, probability and statistics, geometry, and mathematic reasoning.

We differ, not so much in the realm of basic academic standards—the ultimate goals set for our students—but in the teaching methods by which those standards are met.

The question of teaching methodology is at the heart of Carol Feinberg-McBrian's thoughtful response (p. 35) to the "Sacrosanctity Versus Science" keynote address in the previous issue of *Effective School Practices* (Vol. 12, Number 4, pp. 24-32). Feinberg-McBrian defends the NCTM Standards from the standpoint of a mathematician and provides a different perspective on math instruction based on her own experience as a student, and as a teacher. She voices some criticisms of Direct Instruction that are frequently made, giving rise to important questions of teaching methodology, the related issues of scripted lessons, and teaching for understanding. The debate continues with a reaction (p. 43) from the editors of this issue.

The other papers in the current issue, rather than focusing on problems, or perspectives, attempt to describe positive examples of math instruction based on empirical research.

Bob Dixon's paper: *Research-Based Guidelines for Selecting a Mathematics Curriculum* (p. 47) articulates six guidelines for those interested in selecting (or developing) quality mathematics curricula. In contrast to the philosophically derived "vision" of the NCTM Standards, the six guidelines are rooted in research. The paper outlines criteria for evaluating instructional materials based on content and design.

The major consideration for content is threefold. Do the materials, focus on *Big Ideas* (key mathematical concepts), prioritizing areas of mathematics that will provide students with an "umbrella concept" that incorporates other mathematical ideas efficiently? The second consideration for content is whether it presents explicit *strategies* for tackling problems. Good strategies can be applied to a broad range of problem types, but are specific enough to be of substantive, practical help to students. The third aspect of content to be considered is *prior knowledge*. Attention to prior knowledge within a curriculum must be both thorough and timely. Exposure to prior knowledge is insufficient; it must be learned sufficiently well by students that it presents no stumbling block when applied to more advanced concepts and strategies. Once learned, it must also be maintained so that students are still secure in that knowledge when the new, more difficult content is introduced.

In the realm of instructional design, criteria for *scaffolding*, *integration* and *review* are provided. Scaffolding refers to the gradual removal of structure provided by the instruction from initial demonstrations and support through independent practice. The goal of scaffolding is to ensure a high degree of student success throughout the acquisition process. Integration refers to the mixing of problem types to ensure that students learn key discriminations and think carefully about alternative processes. Appropriate review is determined not just by quantitative considerations, but also qualitative; review should be (a) distributed over time, (b) cumulative, with problem sets representing an accumulation of problem types from the beginning of the instructional sequence, and (c) appropriately varied to facilitate generalization.

The final section of this issue provides five research studies that demonstrate the powerful impact these guidelines can have when built into a mathematics curriculum and implemented consistently. Three studies deal with the SRA *Connecting Math Concepts* program, two with the Systems Impact mathematics videodisc programs.

A research brief (p. 62) summarizes the math data from a study conducted by George Brent and

Nicholas Di Obilda in Camden, NJ. They compared the performance of second-grade students taught with the *Connecting Math Concepts* (CMC) program with that of students taught with a curriculum specifically aligned to the CTBS. They were interested in the effects of the instructional programs and also the effects of student mobility, which is particularly high in the Camden schools.

On the CTBS and MAT computation subtests, the CMC students outperformed the control students. On the concepts subtest of the CTBS, the stable control students outperformed both CMC groups (mobile and stable), who in turn performed higher than the mobile control students. The curriculum alignment to the CTBS appeared to benefit only students who were taught with aligned math program for 2 years (30% of the students in the aligned condition). A dramatic contrast was found with the MAT test scores (the MAT was administered to stable students only). CMC students outperformed control students in both computation and math concepts. The results suggest that the curriculum alignment did not result in generalized math abilities for the control students. Performance of the DI students was more consistent across tests, and less imparted by mobility.

The Vreeland et al. article (p. 64) describes the pilot implementation of *Connecting Math Concepts* in third- and fifth-grade classrooms. The authors made several interesting comparisons between the CMC students who had either more traditional basal instruction, or extensive hands-on and manipulative activities. CMC students typically outperformed students of a similar (low) socio-economic status (SES), and performed at comparable levels or higher than students of a higher SES on two standardized and problem solving tests. Encouraging results were also found for gifted students, who made substantial gains and performed well above grade level.

John Wellington's report on the adoption of *Connecting Math Concepts* in a Pennsylvania School district (p. 70) offers a sensitive look at the issues involved in a wide-scale implementation of a Direct Instruction program. The report describes the thorough groundwork that was laid through a pilot implementation, with a well designed analysis of the results. This careful approach, and the success of the pilot implementation, particularly in fourth grade, was instrumental in the subsequent district-wide adoption. This ambitious endeavor has been a significant challenge, and has brought many issues to the fore, including teacher expectations, fidelity of implementation, and resistance to change.

Kitz and Thorpe (p. 76) working with remedial young adults learning algebra, found significant differences in the performance of groups learning from a Direct Instruction videodisc program and from a traditional text. Not only did the videodisc group outscore the text group on the videodisc posttest, but also on the text posttest, despite the fact that the text group scored significantly higher on the test prior to the study.

An equally remarkable finding occurred in the Grossen and Ewing study (p. 79). In the first phase of the study, sixth-grade students classified as "low performers" based on their pretest scores in the videodisc group outperformed the "high performers" in the basal-text group on the basal test. In the second phase, another group of sixth-grade students were more proficient overall on a series of problem-solving tests, when compared with students taught by a teacher expert in the NCTM teaching methods. Such results attest to the power of effective curriculum design and instructional practices, both with remedial and regular education students, resulting in generalizable mathematical knowledge.

In short, there is a sound research base for effective instructional practices in mathematics. Current reform movements have a tendency to ignore this research base and head off in the direction of speculation and conjecture. In the papers that follow, we hope to articulate more fully the nature of this problem, and to offer a more rational, alternative approach.

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What If Those Math Standards Are Wrong?

Chester E. Finn, Jr.
The Edison Project

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Even in the faddy world of K-12 education, the "standards" issued by the National Council of Teachers of Mathematics (NCTM) have met with rare acceptance. Seldom has so profound a change in conventional wisdom and standard practice had such homage paid to it, so little resistance shown to its onrush, so few doubts raised about its underpinnings. Republican and Democrat, textbook publisher and test maker, governor and businessman, federal official and local school board member—just about everyone is rushing to implement "the NCTM standards." What's more, they're ceaselessly cited as the example *par excellence* of what national education standards should look like. Gov. Roy Romer of Colorado, for example, has said so hundreds of times, as has outgoing U.S. Secretary of Education Lamar Alexander. We've seen national, state, and local groups, struggling with standards in their own domains, cite the NCTM, embrace the NCTM and yearn to emulate the NCTM.

We'd better hope the NCTM has got it right. If not, American education's lemming-like rush to follow its lead could find us hurtling off a precipice.

Worries about the NCTM approach to math began to stir in me several years back. They took two forms then; I've lately added a third.

The first had to do with what I've come to realize is vast *misunderstanding* of what the NCTM has actually wrought. It is centered in confusion between "content standards" and "student-performance standards"—to borrow the terminology of the National Council on Education Standards and Testing, co-chaired by Mr. Romer.

Oversimplifying only a bit, *content standards* describe what schools should teach and—presumably—their pupils should learn. Examples might be "state capitals" in grade 4 and diagramming complex sentences by grade 7. Content standards are about curriculum: its goals, frameworks, scope and sequence, etc. They are intended mostly for educators.

Student-performance standards are something else. They involve how well youngsters must do in order to be said to have met the expectations of the content standards. How many state capitals must that 4th grader actually know? Must she have them memorized or is it okay to match up states and capitals from two lists? If she only gets 42 of them right has she fulfilled the standard? Must all those names be spelled properly? As for diagramming, just how intricate a complex sentence do we have in mind? How many must the student diagram? How many errors are acceptable? Such are the issues we must resolve when we set student-performance standards. Note, though, that only when *these* standards are in place can students (and parents) see how well they—and their schools—are doing vis-a-vis what's expected of them. Student-performance standards are truly about results and outcomes. They're meant mostly for laymen.

To date, what the National Council of Teachers of Mathematics has provided are content standards only. Educators are properly grateful, as are many policymakers. But what most nonprofessionals have in mind when they talk about education standards are student-performance standards and those—regrettably—the NCTM has not yet given us.

So how satisfactory are the content standards? We now come to my second worry. It flickered to life when I began to hear of elementary school classrooms where teachers were passionate about "problem solving" but where students were counting on their fingers as late as 3rd or 4th grade because they hadn't learned the rudimentary "math facts" to the point where these came automatically. I contrasted this with what I knew of Japan's "Kumon" math program and with the teaching strategies developed by John Saxon, an eccentric textbook author who is shunned by the math establishment but whose pupils seem adept both in basic arithmetic and in solving complex math problems. (The math establishment despises Kumon, too, of course.)

Was it possible, I asked, that children taught according to NCTM standards might have all sorts of imaginative ideas about tackling a problem yet seldom get the right answer to it because five times 11 was beyond their ken?

Math sophisticates pooh-pooh my concern, arguing that of course the NCTM intends both results, that arithmetic and accuracy are not being sacrificed on the problem-solving altar, indeed that these will develop hand-in-hand.

That, of course, is what everyone yearns for: deft skills and reliable "math facts" *combined* with imagination and deep understanding. And in the hands of terrific math teachers, that's pretty much what happened long before the NCTM was heard from. But U.S. schools don't boast a surfeit of such teachers, especially in the early grades, and in trying to compensate for this kind of shortage (not just in math) we've tended to lurch from one extreme to another, grabbing for the latest miracle cure, forcing tradeoffs, opposing "a" to "b" rather than melding them. For a vivid example drawn from another field, look at the endless war between the "whole language" crowd and the partisans of "phonics," notwithstanding ample research showing that *both* are needed for young students to read effectively and enjoyably.

It's vital to ask of any new approach being thrust upon the education world whether it has been fully tested with students to insure that it yields the desired results.

Great teachers do use both in language-arts lessons, just as they attend both to problem-solving strategies *and* rapid calculation of the right answer in math class. They know these fierce arguments involve spurious choices and phony tradeoffs. But what happens when their professional association slips—or is perceived as slipping—over to one side? In particular, what happens to millions of children whose less-than-gifted instructors rely on prepackaged programs, the latest nostrums, and what others tell them is the approved way to proceed?

So long as many teachers are dependent in this way, it's vital to ask of any new approach being thrust upon the education world whether it has been fully tested with students to insure that it yields the desired results—and is not just being promoted because it appeals to grownups caught up in ideological battles.

Which leads to my third and newest anxiety about NCTM math, seeded by a flawed but compelling (and widely ignored) book by Siegfried Engelmann call *War Against the Schools' Academic Child Abuse* (Halcyon House, 1992).

Professor Engelmann, of course, is the father of dozens of instructional programs, especially for the primary grades, the best known of which are DISTAR reading and math. He is also one of the world authorities on "direct instruction," a highly structured approach that relies on clear expectations for teachers and students, tight performance requirements, "behavioral" (rather than "developmental") instructional practices, and strong emphasis on accountability for results.

Here's what Mr. Engelmann has to say about math à la NCTM:

> The *Standards* de-emphasize anything teachers have failed to teach ... The most serious problem with the *Standards*, however, is its arrogance. In the tradition of the sorting-machine, it assumes that it can derive a curricular reform through metaphysical masturbation of words, not through experimental evidence about what works and what doesn't. The writers of the *Standards* did not verify these activities, suggestions, and standards by first demonstrating that they worked and that they created kids who performed well in math. Instead, they made it up and then presented it as an authoritative document.

If Professor Engelmann is right, we may be buying a pig in a poke, a radical yet unproven overhaul of math curriculum, instruction, and assessment that massages the nerve centers of the "math community" but won't necessarily produce more numerate young Americans.

Mr. Engelmann asserts that the NCTM approach has a lot in common with the debacle known as "new" math. "The manipulatives, the exposures, the acting-out, and the moral insistence on problem-solving," he writes, "has been a theme of math educators since the mid-60s. The approach is actually one of the reasons kids currently don't know long division and are not proficient at paper-and-pencil work in math."

We err when we slight the acquisition of facts, specific knowledge, and simple skills, both as building blocks of more complex intellectual structures and as potent motivators.

The kind of instruction that Mr. Engelmann favors—direct instruction—isn't popular with today's educators. It smacks of rote learning, drill and practice, even memorization, thus of a "canon" of skills and knowledge that every teacher should impart and every pupil acquire. This is unfashionable. It's not what we find in the NCTM standards. But it's performance-oriented, hence amenable to assessment—including the kinds that emphasize *right answers* and thereby lend themselves to accountability, high stakes, and other such scorned practices.

Unfashionable to be sure. But we err when we slight the acquisition of facts, specific knowledge, and simple skills, both as building blocks of more complex intellectual structures and as potent motivators. Many teachers and parents can attest to the satisfaction that kids get from knowing things: precise, definite things that they know they know, can tell they're good at, and from the accumulation of which they can gain a sense of steady progress—in contrast to the subtleties and ambiguities that experts favor. As an example, I recently observed the ardor, pride, and feeling of accomplishment palpable in an elementary school in the much-afflicted South Bronx, a school that is using E.D. Hirsch's "core knowledge" program.

> *Diverse classroom strategies should be welcome—so long as solid learning occurs.*

E.D. Hirsh isn't the main point, though, nor is Siegfried Engelmann, nor even the NCTM. What's important is whether U.S. youngsters actually reach higher levels of skill and knowledge. As yet—a full decade after the National Commission on Excellence in Education labelled us a "nation at risk"—there's scant evidence that our reform strategies are working. The cures we've tried have done little to boost outcomes. To that glum news some people respond by seeking (as often before) to ease the press for results and go back to indices of input, effort, and intention. Others want to replace the measuring sticks, hopeful that different assessments will reveal—and perhaps stimulate—better results from today's voguish curricular and pedagogical strategies. A few, however, are turning away from those strategies themselves, returning to what Marilee C. Rist, in a useful article in *The Executive Educator*, terms "learning by heart." Memorization. Direct instruction. Recitation. Plenty of practice. And gobs of core knowledge.

We oughtn't dump all our eggs into that basket, either. Or any other. No single container is capacious enough. Diverse classroom strategies should be welcome—so long as solid learning occurs. The reason for standards isn't to impose a regiment of what Diane Ravitch terms "pedagogical imperialism." Rather, it's to be clear and prescriptive about ends—and then laid-back and versatile about means.

> ***By focusing on* content *rather than performance standards, the organization [NCTM] has probably led its members and followers to dwell overmuch on what happens in the classroom instead of the results attained there.***

I doubt this was intended, but the NCTM may have given a boost to such imperialist tendencies in math. By focusing on *content* rather than performance standards, the organization has probably led its members and followers to dwell overmuch on what happens in the classroom instead of the results attained there. "Problem solving" works in some situations, to be sure, but "learning by heart" may accomplish more in others. Usually both are vital. Teachers must feel free to adapt their strategies to specific situations, not harnessed to a single pedagogical approach.

Says Thaddeus Lott, the maverick principal of the Wesley School in Houston, an institution attended by hundreds of "at risk" youngsters, a place where DISTAR is used in both math and reading—and where test scores are soaring: "You don't send a guy to dig gold without the proper tools; and you don't build a house without a saw and hammer." By giving his children the tools they need, he is empowering them to build all sorts of structures. But it takes courage to stand up to conventional wisdom. And today that wisdom insists that the NCTM and its ilk have things figured out just right and that everyone had better do things their way. What if they turn out to be wrong?

Divide and Confound

Jay Mathews
Correspondent for *The Washington Post*

Reprinted from The Washington Post, *May 23, 1993.*

Meet the New, New Math. Just Like the Old New Math—Totally Baffling

Scarsdale, N.Y.—I first saw the look on my wife's face—a subtle blend of alarm and irritation—when we were talking with our daughter Kate, 8, about one of her friends. "She's three years younger, right?" Kate said. "So when I'm 12, she'll be 10."

We've had a few more of these arithmetic mishaps since. Kate attends second grade at the Fox Meadow School in Scarsdale, a cheerful collection of long halls and sunny classrooms recently cited by *Redbook* magazine as one of the best public schools in the country. She reads and writes far better than I did at that age and has developed a self-confidence that is usually a comfort and a delight.

But her attempts at addition and subtraction are often hit and miss and awaken bad memories.

> ***Those who remember the famous battle over the New Math of the 1960s ought to be warned that another struggle has begun in America's elementary schools about how to teach mathematics, and the initial signs are not all good.***

Kate, her mother, and I have bumped hard into one of those defining moments in American pedagogic history when all the best ideas about how to help children learn are put to the test. Those who remember the famous battle over the New Math of the 1960s ought to be warned that another struggle has begun in America's elementary schools about how to teach mathematics, and the initial signs are not all good.

The New Math movement, a heartbreakingly ambitious attempt to help children understand mathematics rather than just memorize it, began 35 years ago with an equal lack of fanfare. Little was reported at first about the creation in 1958 of the School Mathematics study Group (SMSG) and a key report by the Commission on Mathematics of the College Entrance Examination Board, but they ushered in a decade of feverish reform and, eventually, dizzying backlash.

> ***When parents discovered that some children were ignorant of multiplication and other practical skills, the New Math crashed with a speed remarkable even for our faddish school system.***

The unifying ideas of mathematics, the reformers said, would henceforth take precedence over drill and rote learning. Grade school teachers began to talk about sets, numeration in bases other than 10 and other topics that mystified parents. The controlling assumption was that every bright-eyed Russian child was already learning this stuff and Americans had to catch up.

Under the traditional programs that teachers were comfortable with, pupils would learn how to add, subtract, multiply and divide by doing basic drills—7 plus 7 is 14, 7 plus 8 is 15, 7 plus 9 is 16—until they were second nature. Under the New Math, students would work with sets and groups of numbers or objects and often leap into simplified discussions of non-base 10 and other untraditional concepts without the drills being fully mastered. Flash cards and other memorizing devices were put away in favor of colorful charts full of arrows and geometric shapes and stories about what fun it would be to live in a world where everyone counted on only two fingers.

Many adults today retain vaguely painful childhood memories of puzzling over their New Math exercises and being as frustrated with them as were their teachers. When parents discovered that some children were ignorant of multiplication and other practical skills, the New Math crashed with a speed remarkable even for our faddish school system. Morris Kline's best-selling book, "Why Johnny Can't Add: The Failure of the New Math" delivered a final blow in 1973, and a back-to-basics movement filled the void.

Why, then, are we back to worrying about simple arithmetic? The reason is that no idea, no matter how battered, ever really dies in American education. Ours is a democracy that believes the schools can solve nearly every problem, and there are always people ready to try something new, or hope that something long ago discredited may have just been ahead of its time.

According to Bruce R. Vogeli, professor of mathematical education at Columbia University's Teachers College, the New Math principle of emphasizing ideas over skills "has been held by mathematicians and educators for at least a century." Felix Klein lectured in the late 19th century on the function concept as a unifying theme. John Perry, Alfred North Whitehead and a host of other educational scholars pursued the idea.

The New Math debacle disappointed them, but by the mid 1980s they could argue that the traditional method was still doing little to raise mathematics scores. The intellectual heirs of Klein and Perry and Whitehead begged for another try at teaching ideas as well as methods. In the last three years they have had enough success in some places to bring parental paranoia up to 1960s levels and make Kate Mathews' parents feel like hopeless mossbacks.

Leading the new reform effort is the National Council of teachers of Mathematics (NCTM), a Reston, Virginia-based organization now at the height of its influence in American schools. University of Missouri-Kansas City professor Shirley A. Hill, a former president of the NCTM, won the $25,000 McGraw Prize in Education for her work on a national blueprint for mathematics education—a 258-page guide entitled *Curriculum and Evaluation Standards for School Mathematics* and usually called the *Standards*. University of Wisconsin professor Thomas A. Romberg, chair of the NCTM commission that supervised the drafting of the *Standards*, called them "a flag to rally around" for teachers who want change. They have become the model for mathematics reform and textbook approval in most states, including Maryland and Virginia, and reformers in other disciplines, such as science and social studies, have borrowed their ideas.

The *Standards* are the work of teams of mathematicians and classroom teachers. The idea is to move students into an age where computers and concepts, rather than multiplication tables and addition facts, rule the mathematical world. The document describes its mission as helping students learn "to trust their own mathematical thinking" and "develop an understanding of mathematical models, structure and simulations applicable to many disciplines."

> ***The [NCTM] document describes its mission as helping students learn "to trust their own mathematical thinking"..... Over pizza recently, Linda asked Kate to add eight plus five. Kate put on an 8-year-old's favorite armor, a look of utter disbelief at the ignorance and misplaced values of adults. "Are you kidding?" she said. "It's 12!"***

The *Standards* pose several problems, however, to classroom teachers, particularly those without mathematics degrees. The guide offers pages of charts, graphs and sample problems, but gives little advice on how to persuade easily distracted students to focus on them. The *Standards* say more time should be spent in teaching problem solving, but do not say exactly how this should be done.

Preliminary research indicates many teachers are dropping drills but not replacing them with anything more useful, an echo of the New Math. Alan Osborne of Ohio State University and Thomas B. Hoffer of Northern Illinois University said they discovered "large declines from 1988 to 1992 in the proportion of students enrolled in classes with a heavy emphasis on learning facts and principles, and on computational skills." If the *Standards* were being followed, the students who are doing less drill work should now be trying to solve sophisticated mathematical problems that reflect the real world. But the two researchers found instead "no increase in the teacher-reported emphasis on problem solving."

Chester E. Finn, Jr., a former assistant secretary of education, wondered in *Education Week* recently if children taught under the *Standards* "might have all sorts of imaginative ideas about tackling a problem yet seldom get the right answer to it because 5 times 11 was beyond their ken." University of Oregon professor Siegfried Engelmann and Doug Carnine, director of the National Center to Improve the Tools of Educators on that same campus, have suggested the *Standards* are still unproven in the classroom.

It is not certain more experiments in sample schools will satisfy either side. Educational researchers have not found a way to turn classrooms into laboratories, with inputs and outputs precisely measured. Tension also seems inevitable between parents and educational researchers, given the way the latter approach their work.

In a 1965 book on the New Math reforms, William Wooton said that "thoughtful members of the mathematical community" in the 1950s felt that high schools had "an unnecessary preoccupation with the immediate usefulness of what was taught." James Fey, professor of mathematics and curriculum instruction at the University of Maryland and a supporter of the *Standards*, said last year "my business is to do research and development on problems of math teaching, and I'm naturally going to be interested in doing things which I hope are on the cutting edge ... and unless I'm out ahead of the schools, I'm not doing my job."

Schaefer said that she is aware that some parents and grandparents have developed arithmetic anxieties. She advised us to relax. "There is a slight delay," she said, "but the kids more than catch up ... and when kids really understand what the facts mean, they can memorize them faster."

Although Vogeli argues that the *Standards* are a much more modest reform than the New Math, he acknowledges that the *Standards'* emphasis on solving sophisticated problems will be a strain on the schools. "It is very difficult to teach," he said, "and teachers and students like to be successful."

The NCTM also faces the daunting task of making its recommendations work in both the best suburban schools, like Kate's, and in poorly equipped urban classrooms with overburdened teachers. Some drafters of the *Standards* were reluctant to try them in disadvantaged schools, but the NCTM resolved to introduce them everywhere. NCTM President Mary Lindquist said, "What we are currently doing, nationwide, is implementing our vision—a vision where all students are given an equal opportunity to learn."

Such ambition is impressive, but not much immediate comfort to those of us watching children struggle with two-digit numbers. Linda, my wife, recently found a set of flash cards and tried those on Kate with limited success. Her teacher, Kathleen Basovsky, assured us that Kate is doing fine and would soon absorb addition and subtraction of numbers up to 20.

The Scarsdale schools, we discovered, use a program called Developing Mathematical Processes (DMP) designed by Romberg and others active in the drafting of the *Standards*. Anne Schaefer, math helping teacher for the Scarsdale elementary schools, said the program is very flexible. Each grade has 14 different units that can be taught in any order the teacher thinks is best.

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Like many wise educators, Schaefer does not discount the importance of tradition and practical skills to both parents and children. When one junior high teacher said some of her students trained under DMP appeared to be weak in computation, more drills were added to the elementary school repetoire.

"Math really should make sense," Schaefer said, "and we are moving away from that shopkeeper's mentality that it is just calculation, but we can move too far, as educators often do."

We will see what happens. Over pizza recently, Linda asked Kate to add eight plus five. Kate put on an 8-year-old's favorite armor, a look of utter disbelief at the ignorance and misplaced values of adults.

"Are you kidding?" she said. "It's 12!"

If learning is so natural, what am I doing in school?

Andrew Nikiforuk
(Title of his new book)

Déjà Vu?—The New "New Math"

Bernadette Kelly
University of Oregon

Abstract: *Educational researchers from the National Center for the Learning and Teaching of Elementary Subjects (NCLTES) observed the extent to which 24 teachers incorporated the* California Mathematics Framework *into their mathematics teaching. The* Framework *recommendations are similar to those found in the National Council of Teachers of Mathematics (NCTM)* Curriculum and Evaluation Standards. *The results reported for four representative teachers raise serious questions about how teachers are supposed to reach the goals of the* Framework. *The first teacher is the most enthusiastic about the* Framework, *having taken a special course on it. An NCLTES author commends her attitude "of exploration and invention, conveying the idea that all students can learn, enjoy, and use mathematics" (Heaton, 1992, p. 155). Unfortunately, the teacher tells her students to multiply length times width to find the perimeter, and to multiply feet times yards when calculating volume. The second teacher is less knowledgeable about the* Framework, *but when she attempts to implement its suggestions, she misteaches averages to her students. The final two teachers seem to produce the greatest student achievement, according to traditional measures. However, these teachers are criticized for not following the* Framework *more closely. Yet the NCLTES authors concede that they are not sure how the teachers should implement the* Framework. *Furthermore, they fail to record what, if anything, the students they observed had learned. Devotion to a new theory that seems to neglect many of the basic needs of teachers and students is not new to education. An earlier and similar theory, "new math," evoked similar devotion and neglect. Unless questioned, this reincarnation may lead to similar disappointments and continued student failure.*

A series of articles in a recent issue of the *Elementary School Journal* has examined how the *California Mathematics Framework* has influenced the teaching practices of fifth-grade teachers in that state (Prawat, R., Remillard, J., Putnam, R., & Heaton, R., 1992). The *Framework* is consistent with the philosophy and recommendations in the *Curriculum and Evaluation Standards* articulated by the National Council of Teachers of Mathematics (NCTM, 1989).

The authors of the articles are professors and teacher-educators affiliated with the National Center for the Learning and Teaching of Elementary Subjects funded by the U.S. Department of Education. They strongly support the reform efforts which emphasize "teaching for understanding" (Prawat, R., Remillard, J., Putnam, R., & Heaton, R., 1992, p. 148) rather than the more traditional emphasis on computation and basic skills. The reform efforts are similar to an earlier movement in math education called "new math." Cowle (1974) described new math this way: "The primary emphasis is on insight and comprehension, *not* meaningless manipulation and reciting by rote. We want thinking, reasoning, and understanding, rather than mechanical responses to standard situations" (p. 71).

The authors described four case studies of teachers selected from twenty-four, chosen to illustrate a range of student populations, and the complex nature of the interplay between teachers' knowledge and beliefs and their responses to the call for change in mathematics education. The first case study (Heaton, 1992) clearly illustrates the orientation that the reformers espouse and how the approach might translate into classroom practice.

Sandra is an energetic, enthusiastic teacher in her fourth year of teaching. Her students are generally from middle-class and upper-middle-class backgrounds. She is knowledgeable about the new *Framework*, having attended a six-week mathematics seminar in a nearby university. She is in firm agreement with the *Framework's* philosophy, goals, and methods for "teaching for understanding." She states, "When I read it, I seemed to be nodding the whole time. I think this is how I teach ...just giving them the formulas and giving them stuff to memorize is worth nothing" (Heaton, 1992, p. 154). Sandra is committed to making math fun and motivating. Heaton praises her approach in this way:

> Sandra's practice illustrates some of the char-

acteristics of a "high-quality" program outlined in the state's *Model Curriculum Guide* (California State Department of Education, 1987). Teachers are to enjoy "engaging in mathematical activities and naturally project an expectation of enjoyment for students" (p. 4). As a teacher, Sandra exhibits an attitude "of exploration and invention, conveying the idea that all students can learn, enjoy, and use mathematics" (p. 6).

Sandra likes to teach mathematics and is identified by both district personnel and a university-based staff developer as someone who teaches in ways consistent with the *Framework*. She is, from their perspective, an exemplary teacher within her school district (Heaton, 1992, pp. 154-155).

A second feature that aligns Sandra with the approach advocated by the NCTM and California *Framework* is her discretionary use of the textbook. She is using an innovative state-adopted text designed to align math instruction with the *Framework*. In her attempts to make math fun and engaging for her students, Sandra frequently departs from the textbook. Teaching directly from a text, she says, "just bores me to tears . . .I don't like teaching the way that they like to teach, because somebody else wrote the book, and they are not me ...it doesn't work. Kids don't get it and I don't get it ...So pretty much I improvise ...I must have skipped 150 pages in this book" (Heaton, 1992, pp. 155-156). Since becoming aware of the *Framework*, Sandra has felt "a lot more comfortable not necessarily going by the book and bringing in other stuff" (p. 156).

Heaton agrees with Sandra's position: "This seems like a responsible way for Sandra to make use of a mathematics textbook—use it as a resource with her students in mind rather than as a prescription. In fact, Ball and Feiman-Nemser (1988) show that this is what some teacher education programs are teaching students to do" (Heaton, 1992, p. 156).

On one of the lessons Heaton observed Sandra teach, students were engaged in designing a park—an activity Sandra learned about at a recent five-day inservice. Heaton contacted the workshop organizer:

> I was especially interested in what mathematical knowledge they thought a teacher would need in order to use this activity. I did a telephone interview and asked, among other questions, "What does a teacher need to know about mathematics to do the park activity with a group of students?" The program staff member with whom I spoke replied, "I don't think a teacher would have to know anything in particular ...the activity itself should carry the mathematics" (Heaton, 1992, pp. 158-159).

Students worked in cooperative groups, selecting materials for the park (e.g., swings, sand, fencing, tables), and keeping materials within a $5,000 budget. Some items (bricks, tables) were priced individually, rope was priced per 10 feet, fencing was priced per 10 running feet, sand was priced per cubic foot.

> ***The question remains: What progress are we making towards world-class standards, even if students are "actively engaged, having fun, feeling successful, enjoying themselves, thinking, and problem-solving in the context of real-world situations " if they are not, in the process, learning correct mathematics concepts?***

When asked if she anticipated any difficulties with the task, Sandra thought that the calculations might prove difficult: "Some things are per feet, and some are each, and some are pairs, and you know, I think that will be hard, that will be a struggle" (Heaton, 1992, p. 159). Sandra was correct. After observing the lesson, Heaton makes the following comment:

> ...moving away from the textbook and developing one's own plans require a flexible understanding of the content to be learned as well as ideas about how to help students learn. Sandra seems to have clear ideas about how children can be helped to learn (the teacher makes the activity fun), but observing her classroom led me to question her understanding of the content (p. 156).

Here are excerpts from Heaton's description of the following day's lesson:

> The calculations were more procedurally and conceptually complex than either Sandra or

her students realized. Students raised the problem of how to calculate the cost of fencing for the park whose size was to be 200 x 300 feet. The fencing was priced at $30 per 10 running feet. Sandra told students to multiply 200 x 300 to figure out the total amount of fencing they needed. She gave students the answer, "60,000 feet." By multiplying the length times the width, however, Sandra had calculated the area of the land inside the fence, not the amount of fencing. The perimeter, found by adding the length of the sides of the park (200 + 200 + 300 + 300), is the measurement needed to calculate the amount of fencing needed. I did not see Sandra or her students try to calculate the cost of the fencing once they thought they had found the amount of fencing needed.... [The cost] would equal $180,000, ...far exceeding the $5,000 budget of the park. Sandra's approach to this problem suggests that concepts of area and perimeter—and the difference between them—were not her focus. One is led to question her understanding of the concepts.

Sandra was also unprepared when the problem arose of calculating the cost of sand, which was priced per cubic foot. Sandra reflected in an interview after class: "Something came up today when we were doing the park and they wanted to know what cubic foot was. You know, the thing is that I couldn't really answer that question. Then I thought and I thought, then I remembered how to measure a cube. You know the area of a cube is height times width times length or whatever. So then, we looked around and looked [it] up in the dictionary, and we put it together what a cubic foot was.... Then we went from there." ...Once cubic foot had been defined, Sandra had the idea that groups should measure a large sandbox on the school's playground.... Two students worked on measuring the length while the others measured the width.... The pair measuring the length reported 46 yards, and those measuring the width reported 10 yards.... Sandra helped them to measure the height. They realized they could not use a yard as the unit of measurement because it was too large. Someone suggested that they use inches. They counted the number of inches and concluded that the height was 1 foot. Now that they had measurements for height, length, and width, Sandra instructed the students to multiply the numbers 46, 10, and 1 together. Forty-six yards x 10 yards x 1 foot equals 460, but it is neither cubic yards nor cubic feet. To make sense of these measurements one must convert them to common units of measurement (Heaton, 1992, pp. 159-160).

Neither the teacher nor her students were aware that any error had been made in the calculation of the length of fencing or the volume of the sand pit. Heaton rightly concludes that: "In this lesson, Sandra's efforts to teach for understanding may have led to misunderstanding" (Heaton, p. 160). As far as Sandra was concerned, however, the activity was a success. Heaton, herself, also stresses the "positive" aspects of the Lesson:

> This is not to say that what guided Sandra's instructional decision making is unimportant. Students being actively engaged, having fun, feeling successful, enjoying themselves, thinking, and problem solving in the context of real-world situations should not be minimized. These are all objectives of the *Framework* (Heaton, 1992, p. 161).

But the question remains: What progress are we making towards world-class standards, even if students are "actively engaged, having fun, feeling successful, enjoying themselves, thinking, and problem-solving in the context of real-world situations " if they are not, in the process, learning correct mathematics concepts?

The second article, a case study of Valerie (Putnam, 1992), presents a similar situation. Valerie, though less familiar with the *Framework*, is an experienced teacher who enjoys teaching mathematics. However, her goal for her students, "to make them prepared for the world of math in everyday life experiences" (p. 164), is seen as inadequate, because, according to Putnam, her view of the math to be applied by her students is "more mechanical and computational than the *Framework*'s view" (p. 164). The *Framework* states:

> Teaching for understanding does not mean that students should not learn mathematical rules and procedures. It does mean that students learn and practice these rules and procedures in contexts that make the range of usefulness apparent (California State Department of Education, 1985, p. 13).

Ironically, when Valerie attempted to do just that—apply rules and procedures in contexts that make the range of usefulness apparent—she also led students into a mathematical situation for which neither she nor her students were prepared.

Putnam (1992) observed two lessons on averages. In the first lesson, students were directed through a textbook lesson to find the average of several sets of numbers representing different situations (e.g., test scores, lengths of jumps in a competition). The procedural steps emphasized were: add, then divide. Putnam summarizes Valerie's approach this way:

> Valerie's emphasis on learning the steps of computational procedures and being able to apply them to problems in everyday life is a reasonable, although limited, perspective. Most mathematics educators, however, including the authors of the *Framework*, envision a more flexible and reflective approach to applying mathematical tools to solving problems (p. 170).

In the second lesson, Valerie departed from the textbook and involved the class in several "surveys" in an attempt to show the application of averages to situations in the students' own lives. The first survey was intended to determine the average number of times students had eaten ice cream over the past seven days. Students reported the number of times they had eaten ice cream in the past seven days. The data was recorded on the board as follows:

[# times]	[# students]
0	14
1	2
2	5
3	4
4	2
5	3
6	0
7	0

To calculate the average number of times ice cream had been eaten, Valerie added the numbers in the second column and divided by the number of categories: $30 \div 8$. The result was 3.7, which Valerie said could be rounded off to about 4. Valerie had inadvertently computed the average number of students in each category, not the average number of times students had eaten ice cream. To compute the intended average, each individual score would need to be added (0+1+1+2+2+2+2+2+...) and the total divided by the number of students (30). Both Valerie and her students were oblivious to the mistake. The remainder of the lesson was taken up with two other surveys involving the number of rooms in students' houses and the size of students' families. The same mistake was made each time. Although he acknowledged the mathematical errors, when reflecting on the second lesson, Putnam (1992) praised Valerie's approach:

> In this lesson Valerie attempted to go beyond the textbook, to provide students with additional practice on the procedure they were learning in the context of meaningful situations. By building the lesson around surveys conducted in class, Valerie illustrated the usefulness and relevance of averages and capitalized on students' interest. Students had the opportunity to practice the two-step averaging procedure presented in the textbook with realistic problems (p. 173).

Putnam (1992) does, however, raise a concern emphasized by the *Framework*:

> [Students] must explore and experiment, ask appropriate questions, and bring forth the mathematical knowledge that will enable them to progress toward answering the questions (California State Department of Education, 1985, p. 3). Students must be actively involved in the processes of problem solving. The teacher should encourage students to think through these processes, foster discussion of ideas and approaches, and guide students to consider the reasonableness of their procedures (p. 15).

And Putnam makes the following statement:

> By emphasizing reflection, teachers might alleviate some of the difficulties created by their lack of knowledge of the mathematics content they are teaching. Had Valerie and her students been more reflective about whether the procedures they were using and the averages they were getting made sense, they probably would have realized that something was not right. Even if they had been unable to find a more appropriate way of averaging their data, they might have realized that applying their unadjusted rule for averaging would not work in this situation, and they might have looked elsewhere for guidance (p. 174).

Perhaps, like Sandra (who was unclear about the meaning of a cubic foot), they would have consulted a dictionary and proceeded with more exploratory "real life" examples. And, as with Sandra, we must ask whether in doing so Valerie's attempts to teach for understanding would lead to further misunderstanding.

> ***Undoubtedly, part of Jim's success stems from his adherence to a "mastery" model of learning: "teach the skill, practice it, apply it, test for mastery, followed by reteaching and retesting of the students who do not achieve the required level of mastery on the first test" .... Stated another way, this approach says: teach what you intend to teach. It fosters accountability.***

The third case study (Remillard, 1992) describes Jim, a dedicated teacher who works primarily with low-income, minority students. Although his students are disadvantaged, and despite Jim's large class size (34 students), his students perform on or above grade level on the California Test of Basic Skills (CTBS) in mathematics. The CTBS tests both computation and problem-solving skills. Remillard acknowledges Jim's efforts this way: "The fact that Jim has been able to get all of his students on or above grade level on this test is a significant accomplishment, considering that students such as his traditionally have low scores on such tests" (p. 192).

Undoubtedly, part of Jim's success stems from his adherence to a "mastery" model of learning: "teach the skill, practice it, apply it, test for mastery, followed by reteaching and retesting of the students who do not achieve the required level of mastery on the first test" (Remillard, 1992, p.192). Stated another way, this approach says: teach what you intend to teach. It fosters accountability. Remillard's (1992) response to this approach is not so favorable:

> Such an approach seems at odds with the *Framework's* focus on understanding and on the development of skills including computational mastery within meaningful contexts. Because these [mastery] materials were developed by the new textbook's publishers, which he understands to be "right with the *Framework*," Jim does not see this mastery approach as conflicting with the *Framework's* philosophy (p. 192).

For Jim, "teaching for understanding means making mathematics applicable to daily life" (Remillard, 1992, p. 185). However, Jim is not merely concerned with test scores. When asked for his views on teaching for understanding versus teaching rules and procedures as contrasted by the *Framework*, Jim has a common-sense response: "You have to have both" (p. 182). "One of our goals, naturally, is to hope they understand why they're doing what they're doing, not just how to do it, but ... you know why you're doing it, then it's of more value to you" (p. 181).

> ***Remillard suggests that "tighter controls" might be needed to ensure that teachers like Jim be brought more in line with the* Framework's *recommendations. But it seems she hasn't paused to reflect whether those recommendations have been shown to result in more effective mathematics instruction than already exists in classrooms such as Jim's.***

Although Jim had not read the *Framework*, this statement seems entirely consistent with its approach:

> Teaching for understanding emphasizes the relationships among mathematical skills and concepts and leads students to approach mathematics with a commonsense attitude, understanding not only how but also why skills are applied" (California State Department of Education, 1985, p. 12).

Remillard (1992), however, sees Jim's approach as inadequate:

> [A] close look at Jim's beliefs and teaching, however, suggests that his interpretation of

> the *Framework* has been shaped by his instrumental nature of mathematics ... Jim interprets the goal of understanding mathematics as being able to apply it to real situations. Consequently, he sees the state's reform efforts as mandating content additions or revisions that fit with his goal of emphasizing real-life application. He thus incorporates aspects of the *Framework*—like estimation—into his instruction without making fundamental changes. Jim does not see the need to revise his goals for students, his views about student learning, or his pedagogical approaches.... One might conclude that Jim's interpretations of the ideas in the *Framework* are distortions resulting from insufficient exposure to the document itself. The job of policy-makers, then, would be to help teachers like Jim learn about the *Framework* more directly. Through reading the actual document, Jim could gain a more thorough understanding of the philosophy and goals that underlie the state's reform efforts that are not addressed in his textbook. Perhaps he needs to attend teacher in-service programs that will help him use the *Framework* in his mathematics instruction. It might also be argued that tighter controls on what teachers teach and how they use their textbooks would result in mathematics instruction more closely aligned to the Framework (Remillard, 1992, pp. 192-193).

Remillard suggests that "tighter controls" might be needed to ensure that teachers like Jim be brought more in line with the *Framework's* recommendations. But it seems she hasn't paused to reflect whether those recommendations have been shown to result in more effective mathematics instruction than already exists in classrooms such as Jim's.

In the final case study (Prawat, 1992), the teacher, Karen, has more than 20 years of teaching experience. She currently teaches at a school where students typically come from upper-middle-class families. Students perform very well in mathematics on the CTBS—typically scoring in the 85th percentile. She is said to represent "... a modest success for the California reform movement" because of her later comments about the relevance of manipulatives, estimation, alternative solutions, and student discourse in math instruction. However, when Karen was initially interviewed, she had serious concerns about the changes implied by the reform movement.

In response to the new math textbook adopted to conform with the *Framework*, Karen protested that in the process of adoption, "teachers don't count" (Prawat, 1992, p. 197). She believes her district likes to go with the

> ...new and fancy.... They say, "Let's look around and see what's new. Oh, we like the idea of high-level thinking skills. You know, that sounds real fancy. Let's go find out about that." And then they go with it without really exploring the whole thing.... They got a little carried away with some of their objectives. They're starting a little too early with a little too much (Prawat, 1992, p. 197).

What motivates this concern, according to Karen, is that the schools are going to end up with children who "can't do any math" (p. 197).

Karen felt her concerns about the new textbook were shared by other teachers: "The first comment out of everyone's mouth that I talked to—and I know of two teachers who piloted it—was that there is too much material in too little time.... They said, 'You'll be lucky if you get halfway through the book here'" (Prawat, 1992, p. 198).

Karen also had misgivings about the emphasis placed on manipulatives in the *Framework*, other reform documents, and the newly adopted textbook. She questioned whether the investment in time for the use of manipulatives paid off instructionally.

> "Those nice boxes of stuff," she said, "are still sitting there." ..."I had to really, seriously say to myself, 'Now, how much did we really get out of this lesson?' because every minute of the time I spend with these kids in here is valuable to me. There just isn't any playtime left" (Prawat, 1992, p. 201).

In her words, the use of manipulatives "is the icing on the cake" (p. 201).

Karen felt there was too much emphasis on "lofty ideas" (Prawat, 1992, p. 198), and that some material should be eliminated, but she was unsure what to drop. "There just isn't time, there just isn't, unless you're going to devote your whole day to math; and at this school, you don't do that" (p. 198).

Karen was pleased to see that the *Framework* acknowledged "Teaching for understanding does not mean that students should not learn mathematical procedures" (California State Department of Education, 1985, p. 13). She said, "I like this statement.... but what I'm afraid happens is the *not* gets left out in our great quest for teaching this understanding—whether it's teachers themselves or the books that lead us to think that you can teach things without any skills" (Prawat, 1992, p. 199). Karen believes that mastering rules and procedures is a prerequisite to application:

> I think we have to be careful that we don't lose sight of the fact that they do need to learn it first.... It really bothers me. Yes. Yes. We're going to solve all these problems without any tools, just by thinking about them. Well, that's totally absurd, if you think about that statement (p. 199).

This view is confirmed by Prawat's observations:

> Occasionally she had to go out of her way, spending more time on certain aspects of the mathematics curriculum until everyone had mastered the content.... Karen seemed to expect such difficulties and appeared willing to persist until they were overcome. She clearly was on the students' side when it came to mathematics" (p. 203).

Karen was clearly well meaning, outspoken, and there is no reason to doubt her competence. But Prawat's (1992) concluding statements about Karen's teaching and beliefs suggest she has fundamental shortcomings as a teacher:

> Karen's views about the nature of mathematics as a discipline do not appear to have undergone substantial change. To the extent that this is also an important issue addressed in the *Framework*, the implementation effort has not greatly influenced Karen's thinking. Mathematics, for her, continues to be a set of tools and techniques; its value lies in its usefulness in helping people meet the daily demands of life. In the *Framework*, there is a reference to "the inherent beauty and fascination of mathematics ...as a subject that can be appreciated and enjoyed by all learners" (California State Department of Education, 1985, p. 1). This implies some appreciation for mathematics as a discipline—as an arena of human inquiry. This perspective is absent in Karen's interview protocols in December and March.
>
> Although one can observe change in Karen's beliefs about the teaching of mathematics as a result of her attempts to come to terms with the new mathematics curriculum—change which, for her, is dramatic and painful—it may not be enough. If teachers are to alter their teaching of mathematics, they may need to reexamine a whole network of beliefs extending far beyond their views about the craft of teaching, narrowly defined; they may need to change their views about the nature of knowledge and how one acquires that knowledge (Prawat, 1992, p. 210).

What exactly do the *Framework* and other reform documents, such as the *NCTM Standards* (and the authors from the National Center for Learning and Teaching of Elementary Subjects), expect of Karen, Jim, and other teachers of mathematics? Would they prefer that Karen and Jim's lessons look like those of Sandra and Valerie? What turns Karen's and Jim's apparent success into failure?

Below are a few excerpts (cited in Putnam, Heaton, Prawat, & Remillard, 1992) that illustrate the authors' views about the shortcomings of current practices and about the "philosophy and goals" that underlie the reform efforts.

Teachers should not demonstrate:

> In classroom after classroom, math has begun with the teacher demonstrating or explaining the particular procedures to be covered in the lesson plans for the day (California State Department of Education, 1987, p. 9).

Teachers should not explain:

> We must see our job as setting up appropriate situations, asking questions, listening to children, and focusing the attention of the students on important elements rather than trying to teach a concept through explanations (California State Department of Education, 1987, p. 13).

Teachers should not teach:

> In reality, no one can *teach* mathematics. Effective teachers are those who can stimulate students to *learn* mathematics.... To understand what they learn, they must enact for themselves the verbs that permeate the mathematics curriculum: "examine," "represent," "transform," "solve," "apply," "prove," "communicate" (National Research Council, 1989, pp. 58-59).

Instead, the teacher provides problem-solving contexts so that:

> ...in working with more complex situations, students will formulate and model problems, screen relevant from irrelevant information, organize information, make conjectures and test their validity, analyze patterns and relationships, use inductive or deductive

> processes, identify or evaluate alternative mathematical approaches, find and test solutions, and interpret results (California State Department of Education, 1985, p. 3).

How these great feats are to be accomplished, however, remains obscure. The case-study authors also seem uncertain:

> It seems premature, however, to claim that we know what the classrooms and teaching envisioned by the *Framework* authors should look like. Strong on theory and vision but weaker on the particulars of practice, the *Framework* suggests necessary changes. The teachers we worked with were left to implement these changes (Putnam, Heaton, Prawat, & Remillard, 1992, p. 215).

What are the authors expecting teachers like Jim and Karen to do? The authors claim that Karen should alter her teaching of mathematics, reexamine a whole network of beliefs, change her views about the nature of knowledge and how one acquires that knowledge. But how does all of this translate into practice? It appears that the authors from the National Center for Learning and Teaching of Elementary Subjects do not know.

And how does all this affect what students learn? It appears that the authors from the National Center for Learning and Teaching of Elementary Subjects do not care. In their discussion of the four case studies, the authors mention, in parentheses: "Because we did not collect student data, we cannot tell what students learned or did not learn" (Putnam, Heaton, Prawat, & Remillard, 1992, p. 217).

One solution to the problem would be to provide teachers with validated textbooks that are carefully designed, presenting the core mathematics content clearly, with careful attention to the concepts presented, their full range of application, and the necessary levels of practice needed for students to achieve uniform success. Such a textbook would need to be followed closely to ensure success.

One must question whether it is ethical to effect tighter controls on how teachers teach, when one is advocating "new" methods that have not been tested and demonstrated to be effective.

These attitudes typify the basic problem with the current reform movement in mathematics. Serious concerns, such as those expressed by Karen, are brushed off with a wave of rhetoric, which do not translate into validated instructional pracices or validated instructional materials. When addressing the problem of content knowledge, Heaton (1992) provides no solutions, but raises some questions:

> How reasonable or feasible is it for reformers, researchers, or teacher-educators to suggest that teachers pay more attention to mathematics content in addition to what they are already doing? Teachers and in-service programs may already have full agendas without attending explicitly to subject matter. Elementary teachers have multiple subjects to teach and affective needs of children to meet, and the time in any in-service program is precious and short-lived. Yet how can thoughtful teachers like Sandra, or thoughtful organizers of in-service programs like the one she attended, promote meaningful mathematics teaching and learning and not pay more attention to mathematical content? (p. 162)

One solution to the problem would be to provide teachers with validated textbooks that are carefully designed, presenting the core mathematics content clearly, with careful attention to the concepts presented, their full range of application, and the necessary levels of practice needed for students to achieve uniform success. Such a textbook would need to be followed closely to ensure success. But this would be unacceptable to the reformers who would have teachers "...use it as a resource ...rather than as a prescription" (Heaton, 1992, p. 156). So while highlighting some of the problems that teachers face, the authors offer no practical suggestions.

Finally, although a relationship between the reformers' mandates and standards and a demonstration of increased student performance has not been established (Bishop, 1990), the authors seem willing to force teachers to adopt these new, vague, untested methods: "It might also be argued that tighter controls on what teachers teach and how they use their textbooks would result in mathematics instruction more closely aligned to the *Framework*" (Remillard, 1992, p. 193).

One must question whether it is ethical to effect tighter controls on how teachers teach, when one is advocating "new" methods that have not been tested and demonstrated to be effective. Yet these calls for reform sound ominously familiar. Rappaport (1976) characterized math education from 1958-1976 as the "new math" era. Macarow (1970) describes new math as follows: "One of the supposed strengths of the new math approach has been in the stress of self-experimentation, self discovery, and minimizing rote memorization while emphasizing the 'seeing' of mathematical structures which lie behind these systems" (p. 396).

Mathematicians believed the new math approach would give students a better understanding of math, but critics like Richard Martin (1973) responded, "There is one slight hitch: Many of these kids can't add, subtract, multiply, or divide." Karen's concern that the schools might end up with children who "can't do any math" (Prawat, 1992, p. 197) might reflect the fact that she remembers "new math" and its aftermath.

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We used to have to teach to *mastery*;
now we have to teach to *mystery*.
Anonymous

Meeting the NCTM Standards through Direct Instruction: *Connecting Math Concepts*

Bernadette Kelly
University of Oregon

Poor performance of United States students in mathematics is widely documented. National and international assessments continue to show that American students leave school ill-prepared for the work force or for future study. A response to the crisis that has received national attention is the document: *Curriculum & Evaluation Standards for School Mathematics*, published in 1989 by the National Council of Teachers of Mathematics (NCTM).

This document sets out an important vision:

- mathematical power for all in a technological society;
- mathematics as problem solving, communication, reasoning;
- a broad-based mathematics curriculum with a variety of contexts and deliberate connections;
- mathematics as an active, constructive process.

The *Standards* also cite evaluation as a key to improving instruction and instructional programs. The purpose of the *Standards* is to facilitate mathematics reform with the goal that "... all students (will) possess a suitable and a sufficient mathematical background to be productive citizens in the next century" (p. 256).

Connecting Math Concepts is a new mathematics curriculum (for grades 1 through 6) that makes definite and positive steps toward accomplishing the NCTM vision. The *Connecting Math Concepts* program carefully develops key concepts inmathematics that equip elementary students with essential and powerful tools and strategies. These tools will enable them to be successful in more advanced mathematical applications.

The program is designed so that each concept that is introduced is developed, extended, and systematically reviewed. Teachers are provided with detailed, carefully sequenced lessons that have been shaped by actual student performance in field-test classrooms. This careful attention to detail provides an instructional program in which lower performing students can learn successfully along with their more able peers.

The *Connecting Math Concepts* program provides teachers with an effective tool to help students tackle real-life problems successfully, see connections between and within mathematical concepts, and develop confidence in their own mathematical ability.

To illustrate how *Connecting Math Concepts* addresses key areas of increased emphasis identified in the *Standards*, the following pages provide some major recommendations taken from the *Standards*, and examples of content and activities drawn from various levels of the *Connecting Math Concepts* program. These examples are samples of a comprehensive, grade-by-grade development of important topics.

NUMBER SENSE AND NUMERATION

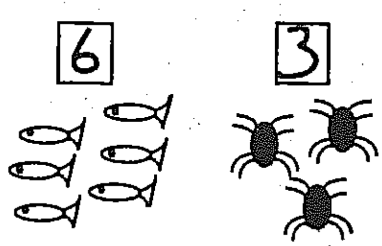
"Counting skills, which are essential for ordering and comparing numbers, are an important component of the development of number ideas. Counting on, counting back, and skip counting mark advances in children's development of number ideas. However, counting is only one indicator of children's understanding of numbers.

Understanding place value is another critical step in the development of children's comprehension of number concepts. Prior to formal instruction on place value, the meanings children have for larger numbers are typically based on counting by ones and the "one more than" relationship between consecutive numbers. Since place value meanings grow out of grouping experiences, counting knowledge should be integrated with meanings based on grouping. Children are then able to use and make sense of procedures for comparing, ordering, rounding, and operating with larger numbers" (NCTM, p. 39).

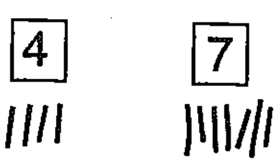
Example of Number Sense: *Connecting Math Concepts* Level A (Grade 1)

Level A of *Connecting Math Concepts* builds on the aspect of mathematics most familiar to young children—counting. Counting skills are extended and used to teach basic mathematical concepts including more, less, place value, and addition.

Relating the counting numbers to the counting of objects is essential for the formation of number concepts. In the beginning lessons, students count a variety of objects, claps and events. They count and write the numeral for groups of pictures:



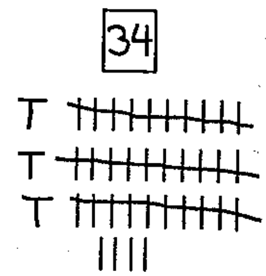
and count and draw lines for numerals:



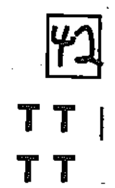
Place value

To record 10 or more counted objects or events, children must understand place value. If there are 3 bottle caps, children write a 3. If there are 30 bottle caps, the children also write 3. The value of the three depends on its position relative to another digit. This is quite an anomaly for many young children—changing the meaning of a symbol because its position changes. Young children have learned that if you change the orientation or position of an object such as a chair, it's still a chair. However, if the position of a digit is changed, its value is also changed.

Place value is introduced by having students write one- and two-digit numerals for objects and for groups of lines. With lines, the children cross out each group of ten and write a T. They can count the T's and leftover lines and write the appropriate numeral:



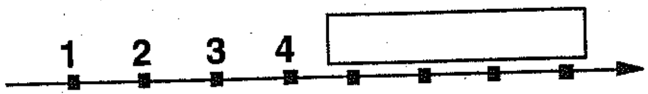
With objects, the students count groups of ten and then count the groups and leftovers. The students also write numerals for groups represented by T's and individual lines:



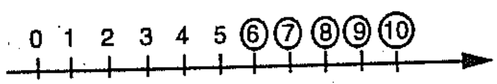
Students also work in the opposite direction, representing numerals with ten and unit counters, or by writing T's and individual lines.

More Than/Less Than

Another basic number concept developed in *Connecting Math Concepts* Level A and based upon counting is relative size. The notion of "more" is introduced in the context of the number line:



In this example, numbers that are covered up are more than 4. Students say the numbers that are *more than 4.* Students then apply the concept of more to objects and pictures. For example, they might make a group that has more than five counters or cross out groups that have more than 7 lines. They also apply the concept to symbols. For example, they might write numbers that are more than 3, or circle the numbers on a number line that are more than 5:



Students also compare groups. First, they identify the group that has more. In symbolic activities, the children circle the number for the greater value.

a. 7 5
b. 9 8
c. 9 10

Students then create the greater-than/less-than sign to indicate which side has more. This activity is carried out with manipulatives (using toothpicks for making the sign) and symbolically. Students are told to make the sign bigger next to the side that has more. Students read inequality statements with the word *less* as well as the word *more.* For example, "6 is more than 4" or "4 is less than 6."

Addition

Counting, place value, and more/less are integrated in the teaching of addition. The concept of more is represented with manipulatives and on a number line. With the number line, children first learn to predict the next number. For example, the teacher covers the numerals after 5 and asks, "What number comes after 5?" Later in the program, children are asked, "What's one more than 5?" The answer is the number that comes after 5 on the number line.

Next, children are asked, "What's 5 + 1?" The answer is one more than 5. The questions are asked in the context of manipulative activities and number-line activities. Once the connection between the concepts of counting, plus 1, and one more has been established, symbolic problems (e.g., 5 + 1= □) are introduced.

Below is an example of an exercise in *Connecting Math Concepts* Level A that incorporates an application of addition and subtraction of larger numbers, the concepts of more and less, and the understanding of one-and two-digit values. Students are to circle possible answers to each problem:

A. 15 + ■ = □ | 16 | 12 | 34 | 9 |

B. 14 – ■ = □ | 6 | 13 | 17 | 1 |

Here are the answers:

A. 15 + ■ = □ | 16 | 12 | 34 | 9 |

B. 14 – ■ = □ | 6 | 13 | 17 | 1 |

Thus, building on their counting skills, students in *Connecting Math Concepts* work with a variety of representations including manipulatives, pictures, the number line, and symbols to develop a thorough basis for number sense and numeration.

MATHEMATICAL CONNECTIONS

"It is important that children connect ideas, both among and within areas of mathematics. Without such connections, children must learn and remember too many isolated concepts and skills rather than recognizing general principles relevant to several areas. When mathematical ideas are also connected to everyday experiences, both in and out of school, children become aware of the usefulness of mathematics" (NCTM, p. 32).

Example of Connections: *Connecting Math Concepts* Level B (Grade 2)

In *Connecting Math Concepts* Level B, money problems connect with many other topics, and provide extensions of numerical concepts (e.g., counting, addition) to problems relevant to everyday experience.

Before Lesson 15, the students learn to count from the numbers 5, 10 and 25. They also learn to identify pennies, nickels, dimes, quarters, and their respective values. They are then well prepared to solve money problems that look like this:

a.  cents

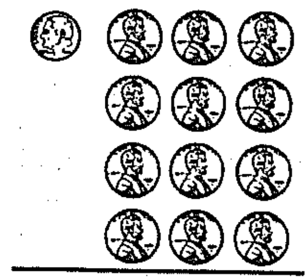
b.  cents

Later, students write multiplication problems for rows of coins. In the example below, the students count by 5 for each nickel. There are 4 coins so they count 4 times.

Students write: 5 x 4 = 20

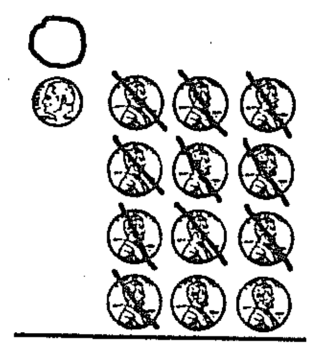
Starting in Lesson 87, students work place value problems that involve pennies, dimes and dollars. In later lessons these problems provide a representation of the algorithm for working column addition problems that involve renaming.

For example:



cents

To work the problem, students cross out ten pennies and draw a dime in the dime column:



2 2 cents

Later the same process for addition is applied to exchanging ten dimes for a dollar. These problems also illustrate the important concept that the same amount of money may be represented with different coin combinations. For example, 10 pennies have the same value as one dime. The concept of coin combinations is further developed by students completing tables that look like this:

					Total coins
a. 65 cents					5
b. 65 cents					4
c. 27 cents					5
d. 27 cents					3

Students use cut-out facsimiles of the coins to represent the amount on each price tag using the specified number of coins.

Once students have represented the amount with the correct numbers of coins, they enter the data in the table. For example, items a and b in the above example would be completed as follows:

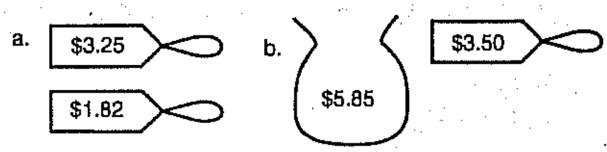
					Total coins
a. 65 cents	2		3		5
b. 65 cents	2	1	1		4

The strategy suggested is generalizable to real-life situations that students encounter when paying for a purchase at a store. An efficient strategy is to pay with as few coins as possible that yield an amount that is equal to, or greater than the cost of the purchase.

Once students have learned to add and subtract in columns, this numerical ability is also applied to the context of money. Students learn to add and subtract money values:

$$\begin{array}{r} \$\,3.48 \\ +\,2.20 \\ \hline \end{array}$$

This knowledge is further applied to problem solving. For example, students work problems that represent objects at a store:



Item a. represents the purchase of 2 objects. To figure out the total cost students write and work the problem:

$$\begin{array}{r} \$3.25 \\ +\,1.82 \\ \hline \$\,5.07 \end{array}$$

Item b. represents having the amount in the bag and spending the amount on the price tag. To figure out the amount left after the purchase, students write and work the problem:

$$\begin{array}{r} \$\,5.85 \\ -\,3.50 \\ \hline \$\,2.35 \end{array}$$

The final money application in Level B involves a table and a series of questions that require column problems.

Example:

	Monday	Tuesday	Wednesday
Rita	$7.00	$ .60	$1.02
James	$2.05	$3.10	$ .62
Sam	$3.29	$ .22	$2.00

a. How much money did Rita earn on all 3 days?

b. How much money was earned on Tuesday?

c. How much money was earned on Wednesday

Through the connections made between money and the mathematical ideas of counting, place value, operations, combinations, and data analysis, students have the opportunity to see the usefulness and connectedness of mathematics.

GEOMETRY AND MEASUREMENT

"Children should have many opportunities to explore geometry in two and three dimensions, to develop their sense of space, and relationships in space, and to solve problems that involve geometry and its application to other topics in mathematics or to other fields" (NCTM, p. 50).

"Geometric models provide a perspective from which students can analyze and solve problems, and geometric interpretations can help make abstract (symbolic) representation more easily understood" (NCTM, p 112).

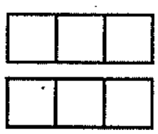
Example of Geometry: *Connecting Math Concepts* Level C (Grade 3)

In *Connecting Math Concepts* Level C, the concept of area is applied to topics other than geometry, and serves as a visual representation for abstract concepts. This section demonstrates the application of a basic geometric concept — the area of a rectangle— to the commutative principle, coordinate system, estimation, column multiplication, and problem solving.

Multiplication

The concept of area builds upon the concept of multiplication—working with equal sized groups.

In this example we have two groups of three blocks.

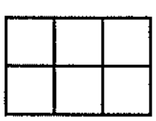


To figure out the total number of blocks students can count by 3, two times: 3, 6.

Students then learn to represent the count–by method as a multiplication equation: 3 x 2 = 6. Counting each block individually verifies that the multiplication method is correct.

Area

By joining up equal-sized groups we can represent the area of a rectangle:



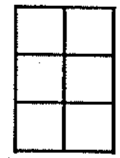
The basic problem is the same as for multiplication: How many squares so we have in all? Students apply the same multiplication method. There are 3 squares in each row and 2 rows, so the multiplication problem is 3 x 2. The area of the figure is 6 squares.

Commutative Principle

The commutative principle of multiplication is visually represented as follows:

Students represent the area of this rectangle as 3 x 2 = 6. The area is 6 squares.

By turning the original rectangle on its side this rectangle is formed. There are now 2 squares in each row and 3 rows, so the multiplication problem is 2 x 3. The area of the rectangle has not changed; it is still 6 squares

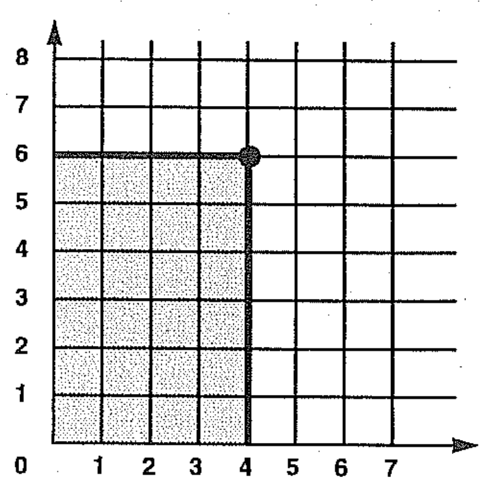


This demonstration represents the equivalence of the multiplication equations: 3 x 2 = 6 and 2 x 3 = 6. This equivalence is true for all rectangles. Students subsequently understand that if they know the answer to 7 x 9, they also know the answer to 9 x 7. Students come to understand that the order of factors may be reversed without changing the product.

Coordinate system

Students apply their knowledge of rectangles to a new context: the coordinate system. When one corner of a rectangle is placed at the origin (0,0) of a numbered grid, the opposite corner may be represented by the coordinates for that point. For example, the point (4,6) represents the corner of a 4 x 6 rectangle.

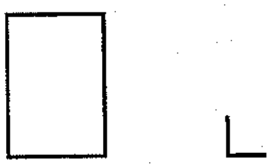
The students use the x and y values as a code for drawing a rectangle. They then figure out the area of the rectangle. The coordinate grid serves as a way to check their calculation by counting the number of squares to verify their answer.



Estimation

Estimation is often difficult for students because they do not have a frame of reference upon which to base their estimation. Without such a reference students are not estimating, but merely guessing. The frame of reference also provides a basis for students to judge the reasonableness of their answers. In one introductory estimation exercise students construct a rectangle 4 inches wide and 5 inches high. They use a ruler to create the 4 inch side, but the second (5 inch) side they draw without using the ruler. They use the 4 inch side as the basis for estimating the length of the second side. They may visually check the reasonableness of their answer (it should be a little longer than the 4 inch side) and also check the accuracy of their estimate by measuring the exact length of the second side they have drawn. They then multiply to figure out the area.

More advanced exercises provide a referent rectangle, and dimensions for a second rectangle, for example:



4 x 6 3 x 7

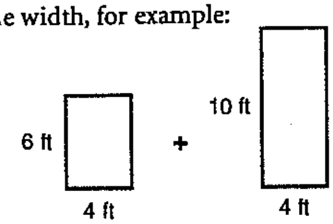
Students estimate both dimensions based on the original rectangle; the new rectangle should be a little narrower and a little taller than the original rectangle. Again, students can visually check the reasonableness of their answer, and measure to check the accuracy of their estimates, before calculating the area. Thus, the concept of area provides a familiar context for the new concept of estimation.

Column Multiplication

The area of a rectangle may also provide a representation for the algorithm used to solve column multiplication problems. For example:

$$\begin{array}{r} 16 \\ \underline{x\,4} \\ 24 \\ \underline{+\,40} \\ 64 \end{array}$$

Students first figure out the total area for two rectangles that have the same width, for example:



Students multiply to find the area of each rectangle, then add to find that the total area is 64 square feet:

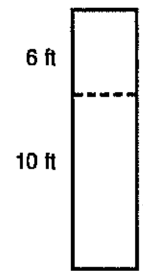
$$\begin{array}{r} 6 \\ \underline{x4} \\ 24 \end{array} \qquad \begin{array}{r} 10 \\ \underline{x4} \\ 40 \end{array} \qquad \begin{array}{r} 24 \\ \underline{+40} \\ 64 \end{array}$$

Students see that column multiplication may be used as a short cut for figuring out the total area of two rectangles with the same width. The width is written only one time:

Students first figure 6 x 4, then figure 10 x 4 and add, just as they did earlier.

$$\begin{array}{r} (10 + 6) \\ \underline{x\,4} \\ 24 \\ \underline{+40} \\ 64 \end{array}$$

Finally, the combined area of the two rectangles may be thought of as one large rectangle:



The entire rectangle is 16 ft long and 4 ft wide. Students work the problem:

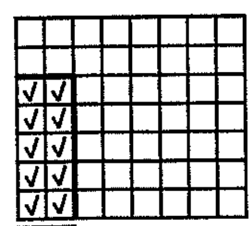
$$\begin{array}{r} 16 \\ \underline{x\,4} \\ 24 \\ \underline{+40} \\ 64 \end{array}$$

The total area has not changed; it is still 64 square feet. A similar analogy could be used for the distributive property of multiplication: $4 \times (6 + 10) = (4 \times 6) + (4 \times 10)$

Problem Solving

Multiplication and division word problems may be introduced using a rectangle on a grid, similar to the rectangle on the coordinate system illustrated earlier. For example: A rectangle has 2 squares in each row. There are 10 squares in all. How many rows are in the rectangle?

Students first draw a line under 2 squares on the bottom of the grid, then count by twos, making a complete row each time they count, until they reach 10. They can then see the number of rows: 5.



Later, students work similar multiplication or division rectangle problems without using a grid. This type of problem lays the groundwork for analyzing and solving word problems that are linguistically similar, but involve a variety of objects or events. For example: Each dog had 5 bones. There were 30 bones in all. How many dogs were there?

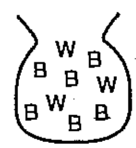
Thus *Connecting Math Concepts* uses the area of a rectangle to provide a geometric model that is applicable to other topics in mathematics, and to help make abstract procedures or concepts more easily understood.

STATISTICS AND PROBABILITY

"An understanding of probability and the related area of statistics is essential to being an informed citizen. Often we read statements such as, 'There is a 20 percent chance of rain or snow today.' 'The odds are three to two that the Cats will win the championship.' 'The probability of winning the grand prize in the state lottery is 1 in 7,240,000.' Students in the middle grades have an intense interest in the notions of fairness and the chances of winning games. The study of probability develops concepts and methods for investigating such situations. These methods allow students to make predictions when uncertainty exists and to make sense of claims that they see and hear. ...Probability is rich in interesting problems that can fascinate students and provide settings for developing or applying such concepts as ratios, fractions, percents and decimals (NCTM, p.109).

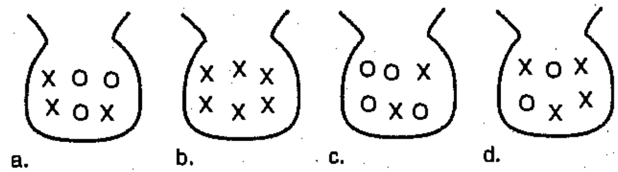
Example of Statistics and Probability: *Connecting Math Concepts* Level D (Grade 4)

In *Connecting Math Concepts*, the concept of probability is carefully developed through an analysis based on fractions and proportions. Consider a bag containing blue and white objects, for example:



The entire set comprises 9 objects. The fraction for the blue objects is $\frac{6}{9}$. The fraction for white objects is $\frac{3}{9}$.

These fractions also represent the *probability* of pulling a blue or a white object from the bag. The closer a fraction is to 1, the greater the probability that fraction expresses. Students in Level D of *Connecting Math Concepts* learn to represent members of a set as a fraction, then apply that knowledge to problems such as:



1. Which bag gives you the best chance of drawing an X?
2. Which bag gives you the second best chance of drawing an X?
3. Which bag gives you the worst chance of drawing an X?

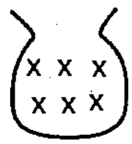
For each bag students write the fraction for the "winners"—in this case, X's. The fraction closest to 1 represents the best chance, and so forth. The fractions also represent the expected outcomes for a given set of trials.

For this bag, the fraction for the X's is $\frac{2}{6}$.

The denominator expresses the number of trials that would yield an expected outcome of 2 winners. In other words, for every 6 trials (shaking up the bag and drawing out an object without looking) one would expect to draw an X from the bag 2 times.

This statement is explored using a bag with a certain number of objects, taking trials, and recording the outcomes. For example, a bag has 7 cards, 4 of which have triangles on them. Triangles are "winners". Tallies are made for each of seven trials taken and for each time a winner is drawn:

Students then see that the expected result does not happen every time, but it does happen most of the time.



For this bag, the fraction is $\frac{6}{6}$, or 1.
A probability of 1 denotes certainty.

If you take trials for this bag, you will draw a winner every time; you cannot lose. It is the only time where you will have a winner for every trial.

Students extend what they know about probability to their knowledge of proportions. This enables them to solve problems such as:

The winners for this set are circles. How many winners would you expect if you took 42 trials?

Students first write the information provided by the composition of the set that gives the ratio of winners to trials:

$$\frac{\text{winners}}{\text{trials}} = \frac{3}{7}$$

Next, they represent the equivalent fraction that tells about 42 trials.

$$\frac{\text{winners}}{\text{trials}}\ \frac{3}{7} = \frac{\square}{42}$$

Students complete the equivalent fraction:

$$\frac{\text{winners}}{\text{trials}}\ \frac{3}{7}\left(\frac{6}{6}\right) = \frac{\boxed{18}}{42}$$

They conclude that they would expect 18 winners from 42 trials.

This groundwork, expressing probability as fractions and proportions, prepares students for a variety of projects involving dice, cards, and coins. For example, in one experiment, students are presented with a bag and are told, "There are 4 cards in the bag. At least one of the cards is a winner (with a blue triangle on it)." Students write the ratio of the various possibilities for winners and trials:

$$\frac{1}{4} \qquad \frac{2}{4} \qquad \frac{3}{4} \qquad \frac{4}{4}$$

Trials will be taken until 24 winners are drawn. Based on the possibilities, students construct ratio equations predicting the number of trials that would be taken to yield 24 winners:

$$\frac{\text{winners}}{\text{trials}}\ \frac{1}{4}\left(\frac{24}{24}\right) = \frac{24}{\boxed{96}} \qquad \frac{\text{winners}}{\text{trials}}\ \frac{2}{4}\left(\frac{12}{12}\right) = \frac{24}{\boxed{48}}$$

$$\frac{\text{winners}}{\text{trials}}\ \frac{3}{4}\left(\frac{8}{8}\right) = \frac{24}{\boxed{32}} \qquad \frac{\text{winners}}{\text{trials}}\ \frac{4}{4}\left(\frac{6}{6}\right) = \frac{24}{\boxed{24}}$$

The total number of trials and winners are recorded until 24 winners are drawn. The actual number of trials is then compared to the expected values for the four possibilities. Students can then accurately deduce the composition of the bag.

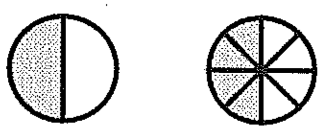
Thus students have a representational framework to make sense of statements such as: the odds are 3 to 2, or the probability of winning is 1 in 7,240,000. Students also see the power of making predictions and drawing accurate conclusions based on empirical observations.

MATHEMATICS AS REASONING

"In grades 5-8, reasoning shall permeate the mathematics curriculum so that students can understand and apply reasoning processes, with special attention to spatial reasoning and reasoning with proportions and graphs.... It is of such great importance that it merits whatever time and effort must be expended to assure it's careful development. Students need to see many problem situations that can be modeled and then solved through proportional reasoning. Such problems can range from simple to complex" (NCTM, pp. 82 and 83).

Example of Reasoning: *Connecting Math Concepts* Level E

Throughout the 5th grade *Connecting Math Concepts* curriculum, students' proportional reasoning is carefully developed through a variety of problem forms. The conceptual development begins with a thorough understanding of equivalent fractions.



$$\frac{1}{2} = \frac{4}{8}$$

In the example above, the fractions 1/2 and 4/8 are equivalent because they represent identical shaded areas for the fraction 4/8. The fractions are proportional because the number of parts in the unit and the number of parts shaded are increased proportionally by a factor of 4. This may be represented numerically as:

$$\frac{\text{parts shaded}}{\text{parts in the unit}} \quad \frac{1}{2}\left(\frac{4}{4}\right) = \frac{4}{8}$$

Multiplying the first fraction by a value that equals one (4/4) validates the fact that the two fractions are indeed equivalent. The fraction 4/8 equals 1/2.

The same proportional reasoning is later applied to simple proportional word problems:

A shop sells special cookies at 10 for $9.00. Tom wants to buy 30 cookies. How much should he pay?

With a solid foundation in equivalent fractions, students can represent the proportional relationship:

$$\frac{\text{cookies}}{\text{dollars}} \quad \frac{10}{9}\left(\quad\right) = \frac{30}{\square}$$

and solve to find the number of dollars needed:

$$\frac{\text{cookies}}{\text{dollars}} \quad \frac{10}{9}\left(\frac{3}{3}\right) = \frac{30}{\boxed{27}}$$ The answer is $27

The same reasoning is also applicable to more complex problems. For example:

The ratio of boys to girls is 4 to 9. If there are 260 children altogether, how many boys are there? How many girls are there?

Students are taught to organize the information logically in a table, showing the ratio values and the actual values in columns:

	Ratio	Number
Boys	4	
Girls	9	
Children		260

They use the additive property of ratio numbers to complete the first column:

	Ratio	Number
Boys	4	
Girls	9	
Children	13	260

They may then operate on two rows from the table to construct an 'equivalent-fraction' equation like those used to solve simpler problems:

$$\frac{\text{boys}}{\text{trials}} \quad \frac{4}{13}\left(\frac{20}{20}\right) = \frac{\boxed{80}}{260} \quad \text{or} \quad \frac{\text{girls}}{\text{children}} \quad \frac{9}{13}\left(\frac{20}{20}\right) = \frac{\boxed{180}}{260}$$

There are 80 boys and 180 girls.

The final applications of reasoning with proportions involve various spatial and graphical representations. Level E includes projects for which students work together in groups or pairs. In one project, the entire class is involved in constructing a proportional model of the solar system. Given information about the earth and the relative distance of the other planets, students first complete a table:

	Distance from the sun				
	Ratio number	Million km	Million mi	Model: Inches	Model: rounded inches
Mercury	.39				
Venus	.72				
Earth	1.00	150	93	10.0	
Mars	1.52				
Jupiter	5.20				
Saturn	9.54				
Uranus	19.18				
Neptune	30.06				
Pluto	39.79				

Students use the same ratio numbers to compute the distance of the planets from the sun, and to figure out proportional distances for a model. Then students work in teams to construct the model along a classroom wall.

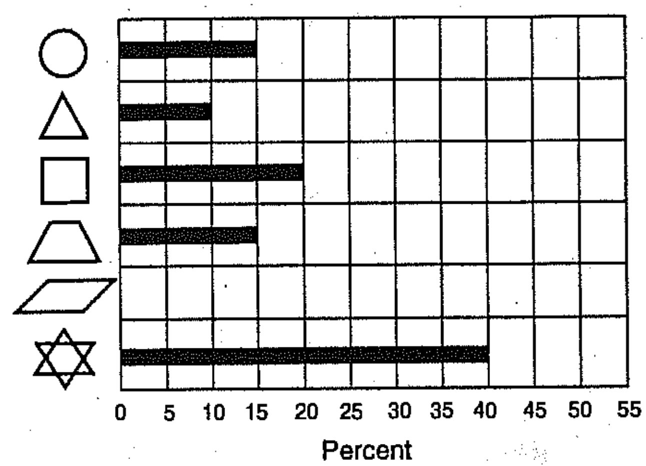
Students also conduct surveys, tabulate the information, and use proportional reasoning to construct a bar graph and pie graph. In another project, pairs of students conduct a survey asking 20 people to identify a favorite shape, and tally results. The tallies will then be converted into percentages (totalling 100%) for a bar graph, and degrees (totalling 360°) for a pie graph, using this table:

	Frequency	Number	%	Degrees
○				
△				
□				
⏢				
▱				
✡				
Total		20	100	360

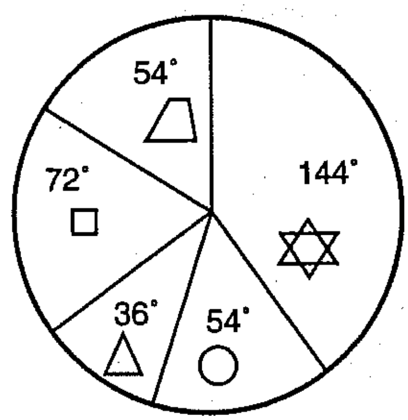
Students then use their knowledge of ratio and proportion to complete the table:

	Frequency	Number	%	Degrees
○	|||	3	15	54
△	||	2	10	36
□	||||	4	20	72
⏢	|||	3	15	54
▱		0	0	0
✡	卌 |||	8	40	144
Total	20	20	100	360

The percents are represented as a bar graph:



The degrees are converted into a pie graph:



The *Connecting Math Concepts* program thus provides a careful development of proportional reasoning. Students apply this reasoning to a variety of contexts, both simple and complex, incorporating spatial and graphical representations.

MATHEMATICS AS PROBLEM SOLVING

"As children progress through the grades, they should encounter more diverse and complex types of problems that arise from both real-world and mathematical contexts. When problem solving becomes an integral part of classroom instruction and children experience success in solving problems, they gain confidence in doing mathematics and develop persevering and inquiring minds. They also grow in their ability to communicate mathematically and use higher-level thinking processes" (NCTM, p.23).

Example of Problem Solving: *Connecting Math Concepts* Levels C-F (Grades 3-6)

The *Connecting Math Concepts* series provides students with an increasing repertoire of problem solving strategies that are fine-tuned to the types of problems they are required to solve. For example, students draw diagrams, make tables, work backwards, and plot relationships.

Diagrams

All single-operation multiplication or division problems may be diagrammed in order to identify the operation required to solve the problem. Consider the following pair of problems:

There are 6 eggs in each carton. There are 12 eggs. How many cartons are there?

There are 6 eggs in each carton. There are 12 cartons. How many eggs are there?

In Level C, students learn the concept of a number-family. For any multiplication/division situation, there are 2 "small" numbers (factors) and a "big" number (product).

A missing big number implies multiplication: $2 \times 4 = \blacksquare$ or $\begin{array}{r} 4 \\ 2\overline{)\,\blacksquare} \end{array}$

A missing small number implies division: $2 \times 4 = \blacksquare$ or $\begin{array}{r} \blacksquare \\ 2\overline{)\,8} \end{array}$

Students learn to represent the values in a multiplication/division situation (one that deals with equal-sized groups) as a number family. The verbal problems above describe groups of eggs in cartons. There are more eggs than cartons, so the total number of eggs is the big number:

$$\overline{)\,E}$$

The number of cartons is a small number:

$$\begin{array}{r} C \\ \overline{)\,E} \end{array}$$

and the number of eggs in just 1 carton is the other small number:

$$\begin{array}{r} C \\ 6\overline{)\,E} \end{array}$$

We can now substitute for the value we know. In the first example there are 12 eggs:

$$\begin{array}{r} C \\ 6\overline{)\,\cancel{E}\,12} \end{array}$$

So the number of cartons is $12 \div 6$. The answer is 2 cartons.

$$\begin{array}{r} 12\ \cancel{C} \\ 6\overline{)\,E} \end{array}$$

In the second example there are 12 cartons.

So the number of eggs is 6 x 12. The answer is 72 eggs.

By representing the situation before choosing the operation, students apply a strategy that helps them analyze the situation and reduces the likelihood of error.

In Level D, students learn that multiplication and division situations may also be solved with a ratio and proportion strategy. For example:

There are 6 eggs in each carton.
How many eggs are in 12 cartons?

Students represent the first sentence as a fraction:

$$\frac{\text{eggs}}{\text{cartons}} \quad \frac{6}{1}$$

The question the problem asks becomes the second fraction in the ratio equation:

$$\frac{\text{eggs}}{\text{cartons}} \quad \frac{6}{1} = \frac{\square}{12}$$

Students figure out the fraction that equals 1:

$$\frac{\text{eggs}}{\text{cartons}} \quad \frac{6}{1}\left(\frac{12}{12}\right) = \frac{\boxed{72}}{12} \quad \boxed{\text{72 eggs}}$$

The same strategy works for division. For example:

There are six eggs in each carton.
How many cartons hold 12 eggs?

$$\frac{\text{eggs}}{\text{cartons}} \quad \frac{6}{1}\left(\frac{2}{2}\right) = \frac{12}{\boxed{2}} \quad \boxed{\text{2 cartons}}$$

Thus, as students develop confidence in solving problems, they also learn that there may be more than one method to solve problems that are equally successful.

Tables

The ability to organize and interpret data in table form is a valuable problem-solving skill. Consider the following problem:

The Army had to ship heavy trucks and light trucks on two ships. The total number of trucks is 344. 137 of the trucks must be light. The rest can be heavy. Joe's ship can carry 59 heavy trucks and 112 light trucks. Belle's ship must carry the rest.

- **a. How many light trucks were on Belle's ship?**
- **b. How many heavy trucks were on Belle's ship?**
- **c. Which ship had more light trucks?**

Students in Levels D and E of *Connecting Math Concepts* learn how to construct a data table using the information given:

	heavy trucks	light trucks	vehicles
Joe's ship	59	112	
Belle's ship			
Total		137	344

Any row or column that gives 2 numbers may be worked by addition or subtraction. Students add to find a total, or subrtract to find a number that is not the total for that row or column.

Students add to complete the top row:

$$\begin{array}{r} 59 \\ +\underline{112} \\ 171 \end{array}$$

and subtract to complete the bottom row:

$$\begin{array}{r} 344 \\ -\underline{137} \\ 207 \end{array}$$

	heavy trucks	light trucks	vehicles
Joe's ship	59	112	171
Belle's ship			
Total	207	137	344

Students may now subtract in each of the 3 columns to complete the table.

	heavy trucks	light trucks	vehicles
Joe's ship	59	112	171
Belle's ship	148	25	173
Total	207	137	344

The answer to each of the 3 questions is found in the table.

- a. 25 light trucks on Belle's ship.
- b. 148 heavy trucks on Belle's ship.
- c. Joe's ship had more light trucks.

Work Backwards

A variety of problem types may be solved by working backwards. Consider the following problems:

- **Start with a number. Multiply by 3. Subtract 20. Then divide by 2 The answer is 8. What's the mystery number?**

- **Jim started out with some money. He spent $45 on sneakers. His mother gave him a gift which doubled the amount of money he had left. Then he found a $10 bill. He now had $40. How much did Jim have to start with?**

These problems can be solved by working backwards because the starting number is the unknown. In Levels E and F students learn an inverse operation strategy. The first step in the strategy is to translate each event into an equation. For example, for the first problem students translate start with a number and multiply by 3 as:

$$\overset{?}{\square} \times 3 = \square$$

The question mark denotes the mystery number. Neither the starting number nor the ending number is known. Next we subtract 20. The ending number of the first equation becomes the starting number for the next equation:

$$\begin{array}{l} \overset{?}{\square} \times 3 = \square \\ \qquad\quad\ \square - 20 = \square \end{array}$$

For the final equation we know that when we divide by 2 the answer is 8:

$$\begin{array}{l} \overset{?}{\square} \times 3 = \square \\ \qquad\quad\ \square - 20 = \square \\ \qquad\qquad\qquad\quad\ \square \div 2 = 8 \end{array}$$

Now students can solve the problem by working backwards. They can figure out the starting number for each equation using inverse operations. They can "undo" division by multiplication, "undo" subtraction by addition, and so forth. First, they multiply by 2 for the last equation. This also gives them the ending number in the middle equation:

$$\overset{?}{\square} \times 3 = \square$$
$$\square - 20 = \boxed{16}$$
$$\boxed{16} \div 2 = 8$$

Next they add 20 for the middle equation:

$$\overset{?}{\square} \times 3 = \boxed{36}$$
$$\boxed{36} - 20 = \boxed{16}$$
$$\boxed{16} \div 2 = 8$$

Finally, they divide by 3 to figure out the mystery number:

$$\overset{?}{\boxed{12}} \times 3 = \boxed{36}$$
$$\boxed{36} - 20 = \boxed{16}$$
$$\boxed{16} \div 2 = 8$$

The answer is **12.**

The same procedure may be applied to the second problem above:

$$\overset{?}{\square} - 45 = \square$$
$$\square \times 2 = \square$$
$$\square + 10 = 40$$

Students work backwards to find the original amount:

$$\overset{?}{\boxed{60}} - 45 = \boxed{15}$$
$$\boxed{15} \times 2 = \boxed{30}$$
$$\boxed{30} + 10 = 40$$

Jim started out with $60.

Plot Relationships

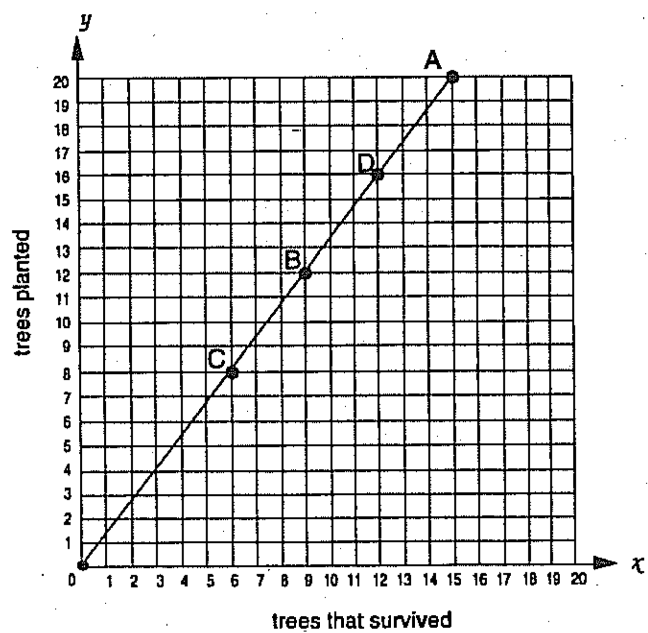
In *Connecting Math Concepts* students work extensively with problems involving ratios and proportions (see the section on Mathematics as Reasoning). In Level E, students also learn to plot proportional relationships on a coordinate system. For example, given the relationship: **For every 4 trees planted, 3 survived,** students first learn to complete a table that gives information about different numbers of trees:

planted	4	20	12		
survived	3			6	12
		A.	B.	C.	D.

Students use their knowledge of ratios and proportions to figure out the missing numbers:

planted	4	20	12	8	16
survived	3	15	9	6	12
		A.	B.	C.	D.

Students then confirm their calculations by plotting each of the points. The numerators are Y values, and the denominators are X values:



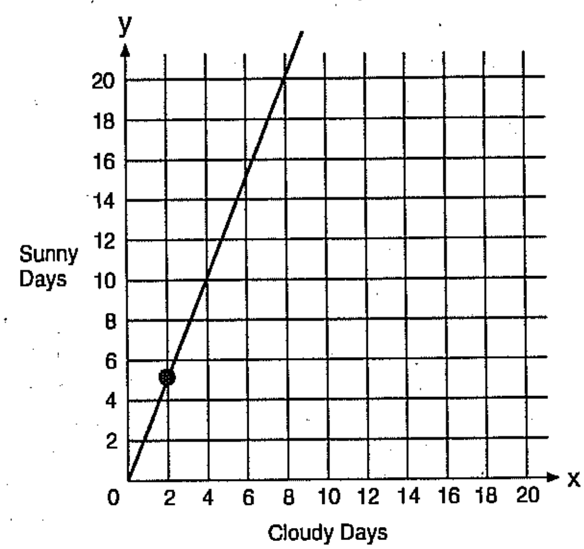
All points should lie on a straight line.

Later, students plot a line based on the given relationship, and use the coordinate system to answer questions.

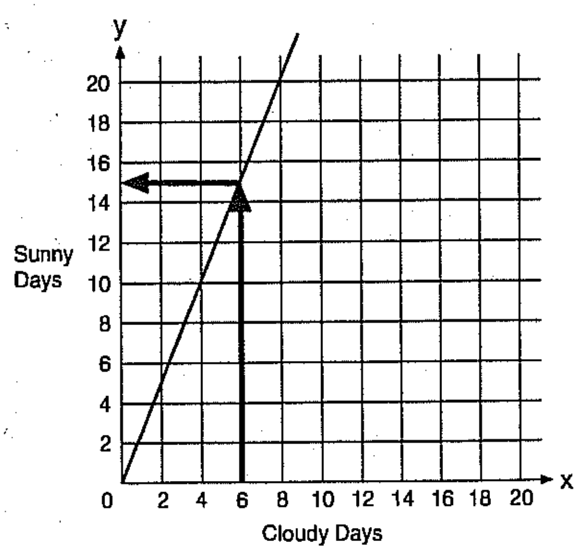
For example:

The ratio of sunny days to cloudy days is 5 to 2.
a. How many sunny days are there for 6 cloudy days?
b. If there are 10 sunny days, how many cloudy days are there?

Students draw the line based on the ratio given:

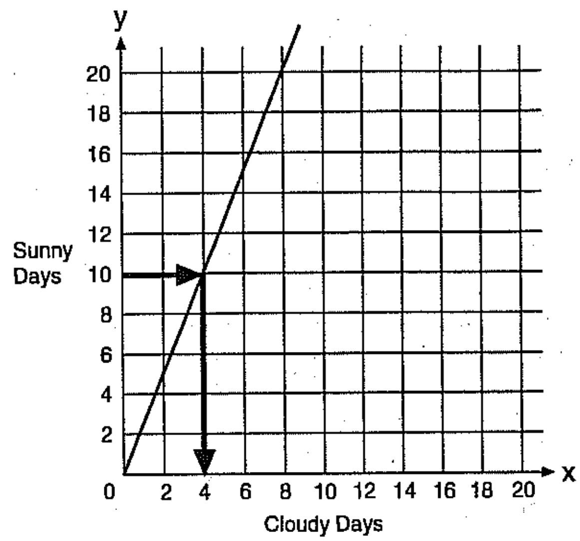


The answers to the questions may be read directly from the coordinate system.



For item a, students find the point on the line that corresponds to 6 cloudy days and read the corresponding value for sunny days (15).

For item b, students find the point on the line that corresponds to 10 sunny days and read the corresponding value for cloudy days (4).



As students progress through *Connecting Math Concepts,* they encounter diverse and more complex problem types that require equally diverse problem solving strategies. These strategies are carefully developed in order to ensure that students experience success in problem solving. They see that different strategies may be applied to work a particular problem type, and at the same time see the interconnectedness of different aspects of mathematics—for example, the operations of multiplication and division, ratios and proportions, and the coordinate system.

CONCLUSIONS

The NCTM sets forth an important vision in a time of crisis. The crisis will not be solved, however, nor the vision realized, unless we implement effective practices and curricula in our schools. The aforementioned examples from the *Connecting Math Concepts* program illustrate the careful approach taken at each level of the program to ensure that students can solve complex problems, reason mathematically, and make connections to the real world. *Connecting Math Concepts* can be a valuable tool for teachers to help their students develop true mathematical power.

A Response to "Sacrosanctity Versus Science"

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Editor's Comment: *This essay is in response to the keynote address by Bernadette Kelly that appeared in* Effective School Practices, *Vol. 12, No. 4.*

There is an old story that attempts to explain the fact that people hold different views about the world. It involves five blind men who encounter an elephant for the first time. Each of the men feels a different part of it in order to find out what it is like. One feels the trunk, another the tail, a third the body, and so on. Naturally, each man's perception of the elephant is different, since each is standing in a different place and holding a different part of the elephant. The men argue violently about their perceptions, each convinced that he alone is correct. The moral of the story, of course, is that, while each has a part of the truth, none of them is entirely correct, since none of them is capable of seeing the "big picture". The only way they could have known the truth would have been for each to listen to the viewpoints of the others and then to find a way to put these observations together in a way that made sense.

This summer, while studying Direct Instruction (DI), I had a chance to see a point of view that I don't usually get exposed to: that of the Special Education teacher. (My usual viewpoint is that of a secondary math teacher.) As one of my assignments, I read Bernadette Kelly's paper "Sacrosanctity and Science" (*Effective School Practices*, Vol 12, No. 4, pp. 24-33). In this paper, Ms. Kelly describes the reactions of several math teachers to her presentations on DI. She found that these math teachers had "assumptions about teaching and learning so different from [her] own that [she] ... had to re-examine ... [her] own beliefs and assumptions about instruction." In doing so, she arrived at the conclusion that these differences are due to a dogmatic attitude on the part of the math teachers and their professional association, the National Council of Teachers of Mathematics (NCTM).

Speaking as a math teacher, I agree with Ms. Kelly's statement that the assumptions behind DI are quite different from the assumptions of the NCTM. However, unlike Ms. Kelly, I attribute this difference, not to any sort of academic dogma, but to the simple fact that the creators of DI and the authors of the NCTM standards have had very different teaching experiences. On one hand, the NCTM is composed of teachers whose primary field is math, and who are intimately acquainted with the skills necessary to succeed in high school and college math courses. On the other hand, DI was developed by teachers working primarily with the naive learner, and its proponents are intimately acquainted with the needs of this population. My experience with DI has not changed my support of the NCTM goals. However, I now also realize the value of DI, especially in regards to the naive learner. My appreciation of DI came, not from re-examining my own assumptions, but from examining the assumptions behind DI.

"We were not communicating," says Ms. Kelly (p. 31) of her exchange with one math teacher. Very true. When people don't understand each others' assumptions, communication can be almost impossible. In this paper, I hope to communicate something of what it feels like to be a mathematician. I also hope to communicate my viewpoint of both the strong points and limitations of DI, and of the NCTM; in a way that will, at best result in better math education; or, at least, create a better dialogue.

First of all, I would like to say that I have learned a lot from DI. Ever since I started teaching, I have been aware of the need to break down skills into small pieces in order for students to understand the lessons. However, my awareness of how to determine preskills and sequence lessons has been vastly improved by studying the DI formats. I consider my DI textbooks (Silbert et al, 1990; Carnine et al, 1990) to contain the best examples of sequencing that I have seen. I have also noticed in my teaching that students tend to confuse new concepts with older ones. It has really helped me to study the DI technique of constantly reviewing concepts and mixing up new problems with the old ones. I believe that this approach is more effective than the more traditional "spiral approach," since it does not give students any opportunity to forget their old skills.

So far, I have mentioned things about DI that almost any good math teacher would appreciate. What most math teachers would not like about DI, and the thing that bothered Ms. Kelly's critics, is the use of the scripted formats. I hope that my audience in the DI camp will bear with me while I try to explain why this bothers us mathematicians so much.

We mathematicians love to think, to reason, to figure things out for ourselves, and to exchange ideas with other students and teachers. For me, and for many mathematicians, one of life's greatest pleasures is to be turned loose with some really strange, new, hard type of problem and to spend days or weeks or even years discussing it and working to solve it. We find passion, joy and intellectual stimulation in mathematics; we long to share this experience with our students. One of the math teachers quoted by Ms. Kelly (p. 29) states that "mathematics is all about ... making discoveries about patterns and structures, and using these discoveries to discover more mathematics". Ms. Kelly seems to think that this is ridiculous. She seems incredulous that this viewpoint is "absolutely consistent with the latest trends in mathematics teaching" (p. 29). However, this is exactly the sort of language that mathematicians use when they try to describe to a non-mathematician what math is all about. Consider these quotes from books written for the general population on the subject of mathematics:

> The joy of mathematics is similar to the experience of discovering something for the first time. It is an almost child-like sense of wonder (Pappas, 1989, front page).

> I've ... learned that mystery is an inescapable ingredient of mathematics ... It's the nature of mathematics to pose more problems than it can solve (Peterson, 1990, xvi).

> Computation involves going from a question to an answer. Mathematics involves going from an answer to a question (Tobias, 1978, 64).

The teacher quoted earlier by Ms. Kelly (p. 29) states that "the child does his or her own learning for themselves ... We need to ask (kids) thought-provoking questions, (and) involve them collaboratively in the discovery". This seems to make no sense to Ms. Kelly. Perhaps she has never had the experience of learning this way, or, if she has, does not consider it significant. It could be that she doesn't observe her own students learning things for themselves because, coming from a DI perspective, she has never given them an opportunity to do so. However, I can testify that discovering or re-discovering things is both the way I learn best, and the way that I prefer to learn. It is the way that my parents, my husband, my children, my most successful classmates, and my most successful students learn. I suspect that it is the way Ms. Kelly's critics learn. Speaking for myself, I'd really hate to learn math by following a scripted presentation, or to be given a page of programmed instruction to do in isolation. It would take all the fun out of math. As a teacher, having to follow a script all the time would deny the reason I teach math.

However, when I look at the needs of the naive learners, I can see that the scripted formats really do have value and purpose. Unlike my colleagues and myself, these students need to have their knowledge given directly to them in small, programmed doses. The formats provide them with the sequencing, constant participation, fast pace, and consistent presentation that they need in order to learn best. Even with sophisticated students, the formats can be helpful in teaching repetitive skills, such as memorizing the basic math facts.

What has made me see the value of the formats is the realization that *they were designed for students who learn differently from me.* I think that it is hard for us math teachers to see the value of DI in teaching the instructionally naive; since we, our colleagues, our children, and maybe even our students are instructionally sophisticated. Even though the results of Project Follow Through, mentioned in Ms. Kelly's paper, have proved DI to be the most effective method of getting disadvantaged elementary students to perform well on standardized tests, we still have a hard time accepting DI, since it is so different from the way that we, ourselves learn. This is our blind spot.

The DI people also have a blind spot. Because they are so used to working with children who need a direct approach, they believe that all children should learn by direct instruction. They assume that because it has proven superior in teaching easily measured skills to disadvantaged students in grades K-3, DI is the best way to teach all skills to all students. Although they would like to believe that the research supports this claim, I see no evidence that it does. In the first place, not all children need a direct approach, Lisa Delpit, an Afro-American research assistant who studies the educational needs of "nonmainstream" children states: "Some children will need to learn explicitly certain strategies or conventions; some children will not need that because they've gotten it ... in their homes" (Delpit, 1991, 544-5). In the second place, there are some aspects of mathematics that cannot be taught by DI. These include the ability to solve new types of problems; the ability to articulate and discuss mathematical insights; and the ability to think of a concept in more than one way. I suspect that the DI people, whose focus is on teaching very specific facts and skills, don't consider these aspects of math to be important. I will attempt to explain why I do.

The first skill I will address is that of being able to form and discuss different mathematical viewpoints. I can tell that Ms. Kelly is skeptical about the value of this skill. When one of her critics writes: "... by questioning, letting children argue, encouraging them to reinvent their own methods of arithmetic ... [we provide] rich mathematical experiences," her response is: "Really? The rhetoric may sound persuasive, but do these practices work?" (p. 25)

Well, it was effective for me. Although my elementary arithmetic classes were very cut-and-dried (and boring), I had parents who valued mathematical exploration, and encouraged me to think about math on my own from an early age. When I got to high school, my ability to think independently about math got me into the "college track" classes, where students could form their own ideas and argue about the validity of these ideas. By the time I was a freshman in college, I was quite confident in my ability to form and support a mathematical idea. So when the professor of my math survey course asked us a question about a certain set of numbers, I raised my hand and gave what I thought was a good answer. He told me (in front of my 300 classmates) that my answer was wrong. However, convinced that I was correct, I went home that night and worked out a proof for my theory. (This took quite a bit of faith, which, in spite of Ms. Kelly's assertion, has its place in math and science.) It turned out that I had un-knowingly discovered something about this set of numbers that was not generally known. In fact, my professor thought that my discovery was important enough for me to publish, although we later found out that someone else had already discovered it.

My mathematical talent did not reveal itself in a classroom setting until I got into high school; as a child, I was so bored by basic arithmetic that I always got poor grades. However, the mathematical habits that enabled me to succeed in my advanced math courses began when I was quite young. In the primary grades, I used to think about things like 1+2=3, 1+2+3=6, 1+2+3+4=10, etc. Although I didn't know it, I was thinking about number sequences. When I got into middle school, I realized that all multiples of three had digits adding up to either 3, 6, or 9. Although I had never heard the term "divisibility rule", that's what I had discovered. When I finally got into algebra, I found, to my delight, (and to my parents' great relief) that I could easily understand abstract mathematical concepts.

My experience with math was not a typical one. A more common pattern is the "sudden death" experience described by Sheila Tobias in the book *Overcoming Math Anxiety* (Tobias, 1978), in which a student who has always succeeded in math by memorizing algorithms suddenly runs into a concept that is hard to understand. Because the student has had no experience with breaking up a hard concept into its component parts, looking at it in different ways, and discussing it with others; he/she has not tools for understanding the concept, and believes that he/she will never understand it. What is worse, the student feels hopeless, not only about the concept, but about math itself. Like my sudden positive experience with algebra, these sudden negative experiences are based on earlier mathematical habits. Tobias (1978, 46) describes the situation of the student who can learn only by memorizing what to do: "Because his grades have been satisfactory, his problem may not be apparent to anyone, including himself. But when his grades finally drop, as they must, even his teachers are unlikely to realize that his problem is not something new, but has been in the making for years.

One of the limitations of Project Follow Through is that, in following the progress of its subjects, it stopped at the sixth grade. There is no information on whether these students were successful in high school math, or whether they had "sudden death" experiences. My guess is, that unless someone explicitly showed them how to think more independently, they were not successful.

The report on Project Follow Through (Silbert et al., 1990, p. 493) states that "[the] tests do not necessarily assess a child's ability to devise unique solutions to complex mathematical problems." In fact, I suspect that DI is unable to teach this skill. I say this, not because of any weakness in DI, but because of one of its strengths: Its rapid pacing. Solving a new type of problem takes lots of time. Mathematicians often work on problems for days, weeks, or even years. In fact, we particularly love to work on problems that have been around for centuries. The ability to do this takes a lot of patience and faith in one's ability to find a solution that is not immediately evident. In fact, it is similar to the definition of religious faith quoted by Ms. Kelly (p. 24)—"the assurance of things hoped for, the conviction of things not seen" (Heb. 11:1). When confronted by a difficult problem, a successful math student has the *hope* that there will be a solution; and has the *assurance* and *conviction* that he/she will either be able to find that solution; even he/she does not yet *see* it.

When children first learn math, they need immediate success, so that they can build up confidence in their abilities. However, once they have some confidence, they also need some experience with finding answers to problems that, at first glance, they had no idea of how to solve. This ability may not seem important to the DI instructor, whose focus is on giving children a good start on their academic careers. However, it becomes more important as the children get older, and are expected to be more responsible for their own learning.

Problem-solving is also an important job skill. Henry Pollak, a respected industrial mathematician, lists these skills among those expected of new employees in industry (NCTM, 1989, p. 4):

> Knowledge of a variety of techniques to approach and work on problems.
>
> The ability to work with others on problems.
>
> The ability to see the applicability of mathematical ideas to common and complex problems.
>
> Preparation for open problem situations, since most real problems are not well formulated.

Ms. Kelly's paper calls for a scientific approach to mathematics. Reading her paper caused me to remember a connection between math and science from my own experience. My father was a research chemist for the Department of Agriculture. It was his job to develop new uses for agricultural products. He loved being a scientist, and made an effort to pass on his scientific values to us kids. When my father talked about science, he did not, like Ms. Kelly, discuss well-designed experiments and statistical results. He talked about the moment of discovery—that moment when the scientist notices something that no one has noticed before. These moments can be dramatic—Marie Curie walking into the lab and seeing the sample of radium glow in the dark— or mundane—Edward Jenner looking at a milk-maid's lovely complexion and making the connection with cowpox. He told us kids to always look for an original answer—to always ask "Why?".

For this reason, he would often tell us mathematical riddles with an unusual twist. A sample was: "A dying sheik left 17 camels to his three sons. Half the camels would go to the oldest, 1/3 to the middle one, and 1/9 to the youngest. They sought the advice of a wise man who helped them divide up the camels without killing any. What did he do?" (Answer: he lent them a camel. The sons got respectively 9, 6 and 2 camels, and the wise man got his own camel back.)

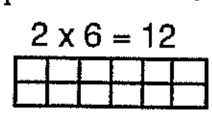
My father's attitude towards math was a typical one for a professional. Research (Donovan, 1990) indicates that, while lower class parents are anxious for their children to learn basic skills, middle class professional ones are more concerned about exploration activities. Perhaps this is, because, like my father, many professionals regularly use this type of skill on the job, while lower-class workers tend to use more basic ones. Since parents tend to pass on their own values and skills to their children, this may explain why DI has been so successful with disadvantaged children. Although disadvantaged children learn successfully with DI, I believe that they also need some experience with less structured problem-solving activities in order to inoculate them against the "sudden death" experience. (Some mentally retarded children may be unable to do this sort of activity. In such a case, I would use only DI.) In giving complex problem-solving activities to disadvantaged children, the teacher must be particularly careful to pre-teach any component skills that the children would need in order to discover the solution, since the children have had no experience with this type of problem. Most math teachers also favor giving students increasingly specific hints about the problem to be solved, rather than the "hmm" response described in Ms. Kelly's paper (p. 30).

An important component skill of finding a new solution to a problem is the ability to see mathematical concepts in more than one way. DI makes a point of using only one model for each process, because the instructionally naive students it was designed for tend to be confused by too many models. However, other types of students often benefit by exposure to different ways of looking at a process. In the first place, the ability to see things in more than one way is helpful, and possibly necessary, to succeed in more advanced courses such as algebra, geometry, trigonometry, and calculus. In the second place, different students have different ways of learning, and the model that makes the most sense to one student might not make sense to another. This was the case with Jason.

Jason is a ten-year-old boy who will be entering the fifth grade this fall. Although he had been in a regular fourth grade, he had not yet memorized the times tables. He had to use a "cheat sheet" whenever he did multiplication problems, and, as a result, also needed to review some of his other multiplication skills. I was tutoring Jason on his math skills, and was also using him as the subject of my DI teaching assignment. One of the skills we worked on was multiplying two-digit numbers by two-digit numbers. When I started to teach him this skill, using the appropriate DI format and the problem 23 x 52, Jason followed me without a hitch. He made a comment that he had done this type of problem before. Everything went well until I made the statement "Since the 5 stands for 5 tens, we will put a 0 in the one's column". Jason's body went stiff. He had an expression of confusion on his face. I had seen this reaction before, and it meant that I was going too fast. At this point, I deviated from my script and said,"Jason, does this make sense to you?" "No!" I departed even further from the script. "Do you learn better when someone just tells you what to do, or when someone shows you why something is true?" "When I know why". I decided that before I tried to teach this skill again, I would first check the format to see if I had done anything wrong. If I didn't uncover any major error, I would switch to a different way of teaching, using a model.

However, before I saw Jason again, I read Ms. Kelly's paper and noticed that the only type of information she seemed to value was research. I though that this was rather unfair, since some teachers, such as myself, really don't have the resources to run experiments, although, by virtue of our different experiences, we might be able to tell Ms. Kelly something she hadn't known before. Then I realized that I did have an opportunity to run a small experiment with Jason. Since he had not responded well to the DI model for two digit multiplication; I would record his reaction to a different model, and then compare the two reactions. Here is what happened:

Carol: Jason, today I want to show you a different way to look at multiplication. Can you see how this picture shows that 2 x 6 =12?

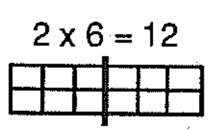


Jason: No.

Carol: There are two rows of squares and six squares in every row—

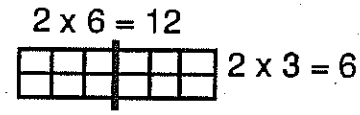
Jason: Oh, wait. Now I see the squares. I've done this before.

Carol: OK, I'm going to draw a line and divide this into two groups of squares. Can you see how this group has 2x3=6?



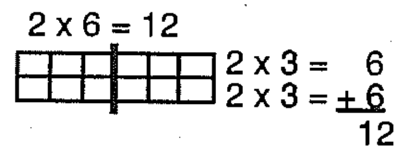
Jason: Uh-huh.

Carol: I'm going to write that down. And this other group has 2 x 3=6?



Jason: Uh-huh.

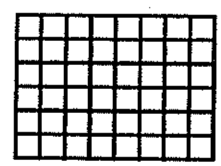
Carol: I'll write that down. Now I can add together the two groups and I get 12. So 2 x 3 plus 2 x 3 is 2 x 6. Does this make sense to you?



Jason: Yeah. I think I've seen that before.

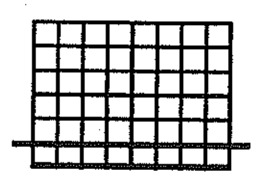
I showed Jason some more examples, always using a fact that he knew for the large rectangle. Now I was ready for the big step.

Carol: We can use this method to find the answer for multiplication facts that we don't know. Tell me what this picture shows.



Jason: That's 6 x 8. (This is a fact that Jason hasn't learned yet.)

Carol: I'm going to show you how to multiply 6 x 8. First I divide it into two pieces. What is this piece?



Jason: 5 x 8 .

Carol: Good. We'll write that down. Now, what is this piece?

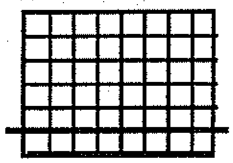
Jason: 1 x 8.

Carol: Good. We'll write that down. Now what do we get when we multiply?

Jason: 5 x 8 = 40 and 1 x 8 =8.

Carol: Right. And what do we get when we add them together?

Jason: 48.



5 x 8 = 40

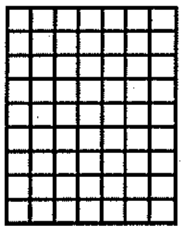
1 x 8 = +8

48

Carol: Right. So what is 6 x 8?

Jason: (Jason's eyes get big.) 6 x 8 is 48. Wow!

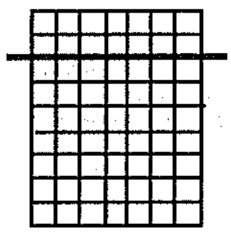
Carol: You can find the answer to any multiplication problem just by dividing the numbers into smaller pieces. This next rectangle shows 9 x 7, but we're going to find the answer by dividing it into 9 x 2 and 9 x 5.



Carol: I want you to draw the line this time.

Jason: Me?

Carol: You. (Jason thinks a while, then draws the line as shown below.)

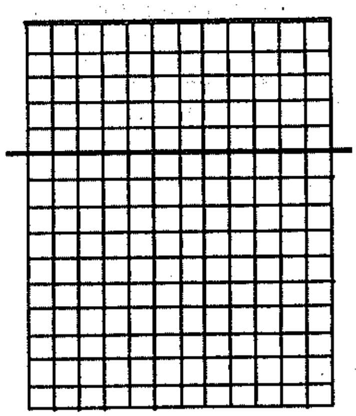


Jason: Ta-DA!

Carol: Good. Now what do we do?

Jason: 9 x 2 is 18 and 9 x 5 is 45. So 9 x 7 is ...56! Verrrrry interesting!

Carol: So now we're ready to multiply 15 x 12. I want you to divide this into 5 x 12 and 10 x 12. (Jason draws the line as shown below.)



Jason: Ta-DA!

Carol: So what happens next?

(Jason does the multiplication for 5 x 12 and 10 x 12)

Jason: So 60 + 120 is 180. 15 x 12 is 180!

Carol: So now if you see a problem like this, you know what to do.

$$\begin{array}{r} 15 \\ \underline{\times\ 12} \end{array}$$

Jason: Yes. It's fun this way. Hey, are you writing down what I'm saying?

Carol: Uh, yes.

Jason: Why?

Carol: Well ... I wanted to tell my teacher what you said when I showed you how to multiply this way.

Jason: Oh. Well, in that case, I want you to write, "oh, cool, oo-wee—cool, wow, mad-bad-I'm totally rad!"

Carol: Great! I'm really glad you like to do this. Now let's do some multiplication by 10. Do these problems.

8 x 10 =
9 x 10 =
10 x 10 =
11 x 10 =

Jason: 8 x 10 is 80, 9 x 10 is 90, 10 x 10 is ... 100, 11 x 10 is ... 110?

Carol: Right. Now, what about these?

17 x 10 =
23 x 10 =

Jason 17 x 10 is ... 170?

Carol: Right.

Jason: Now I get it! 23 x 10 is 230.

Carol: Yes! You've got it! Now, how about these?

20 x 6 =
20 x 11 =

Jason: 20 x 6 is .... Would that just be 120?

Carol: Yes. You've got the idea.

Jason: So 20 x 11 would just be 220.

Carol: Yes. That's it.

Jason: I get it now.

We returned to the two digit by two digit problems. This time, Jason had no problem with the zero in the one's place. We spent a total of 15 minutes on the whole process, and by the end of it, we were both full of energy and excitement. My hypothesis had been that Jason would learn better and enjoy it more if I used a model that showed why the algorithm for two digits by two digits works. I think that the data overwhelmingly supports my hypothesis.

Although I deviated from the DI format while teaching Jason this particular algorithm, I still found that DI worked very well for Jason when he was learning his multiplication tables and his basic division skills. I am sure that Jason learned more from the combination of techniques than he would have by any single method.

Jason is not the only student with whom I have used DI in combination with less structured methods. This summer I had three other tutees in addition to Jason. I found it very easy to switch back and forth between direct teaching and indirect teaching in response to the needs of my tutees.

I have been asked how I decide when to use a particular method of teaching. The answer is that I look at the nonverbal, verbal, and written responses of my students. For instance, with Jason, I followed the DI format until I noticed that Jason's body language indicated distress. Then I followed up with some direct questions. Jason's answer that he liked to know "why?" told me that he might respond well to a more direct approach. When I tried the new approach, I kept asking him whether it made sense to him. When the answer was yes, I proceeded with the lesson. When it was no, I gave a further explanation. As the lesson progressed, I had Jason draw lines and write down calculations to make sure that he was understanding me. Finally, I noted his enthusiastic verbal responses and his more comfortable body language as he started to get interested in the lesson.

I am not the first teacher to combine DI with other methods. As part of my work for a class in teaching reading by DI, I read an article by Dixie Lee Spiegel (Speigel, 1992) telling how several reading teachers have combined DI with Whole Language in a way that gives their students the benefit of both approaches. Like me, these teachers find it easy to modify their approaches to fit the needs of the moment. Lisa Delpit says of her work with "nonmainstream" children, "I don't place myself as a teacher in a camp. I see myself as a responder to the needs of children" (Delpit, 1991; p. 544). Delpit finds that "instruction can be both child-centered and teacher-directed at the same time. The child-centeredness comes with understanding what the child needs at any given moment. Sometimes a particular child might need direct instruction" (Delpit, 1991; p. 547). But sometimes a child may need a different approach. A good teacher should be prepared to meet both of these needs.

Although some teachers are already combining direct and indirect instruction, others feel comfortable with only one way of teaching. These teachers would probably improve their ability to respond to the needs of all their students by learning something about another style of teaching. Teachers who, like myself, come from a process-oriented background can gain a deeper understanding of how to reach their slower learners by studying DI. Even if they decide never to use an entire format, they will have the benefit of really concentrating on sequencing, reviewing, and involving their whole class. On the other hand, teachers who know only the direct approach would benefit from taking a class, such as Math 105, in which students can experience problem-solving and discussing their mathematical ideas in a relatively non-threatening situation. As they get more comfortable with these processes, they may decide to devote a portion of their time to more discovery-based activities. Even if they don't, they will learn how to value and encourage the child who always asks "why?" Once teachers have an idea of how both types of learning work, it should not be hard for them to find a way to combine them. Ms. Speigel notes that "the reality of the world of schooling is that teachers *will* draw what works best for them from both worlds (Speigel, 1992; p. 43).

Ms. Kelly feels that what math education needs is a "unified scientific perspective" based on research. I disagree. Research won't do one bit of good unless people make use of its results. And people won't make use of research, no matter how good that research is, if it seems to contradict their deepest values. Ms. Kelly's critics are well aware of the research showing the benefits of DI. Yet, they are no more willing to see any value in DI than Ms. Kelly is to see any value in their insights about mathemathics, because the values of the two sides seem to be so contradictory. When I started my courses in DI, I was aware of the research, but I did not think that DI could be relevant to my own teaching. Now that I understand the purpose behind DI, I consider it extremely relevant to me, because it showed me how to understand students who are very different from myself. Now that I can look from both viewpoints, I can see that, rather than contradict each other, they complement each other.

One of the goals of both the DI people and the authors of the NCTM standards is to provide all children with the skills needed to succeed in mathematics. Neither group will be able to accomplish this task without acknowledging the insights of the other. Without the careful sequence, review, and participation provided by the DI formats, the NCTM will be unable to reach the most disadvantaged of our nation's children, and will miss a chance to give every child a solid start in the basic skills. But without the NCTM's emphasis on problem-solving and mathematical community, the DI people won't be able to inspire the mathematically creative child, or help students develop independent thinking needed for success in high school and college math courses. In order to develop a realistic vision of math education, we have to learn to see from both perspectives.

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Reaction to Feinberg-McBrian's Comments

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University of Oregon

Carol Feinberg-McBrian offers a thoughtful and provocative response (pp. 35-42 this issue) to the keynote address: "Sacrosanctity Versus Science" (Kelly, 1993). We would like to respond with a few points of clarification.

Means Versus Ends

We also agree with the NCTM goals that all students should be equipped to reason mathematically, communicate mathematics and solve problems. They need to be equipped to be "productive citizens in the next Century" (NCTM, p. 256).

Feinberg-McBrian describes herself as a mathematician, and explains:

> "We mathematicians love to think, to reason, to figure things out for ourselves, and to exchange ideas with other students and teachers. For me, and for many mathematicians, one of life's greatest pleasures is to be turned loose with some really strange, new, hard type of problem and to spend days or weeks or even years discussing it and working to solve it" (p. 36).

No doubt this is true of mathematicians. We have no difficulty accepting this as a valuable goal for mathematics instruction. However, we do have difficulty with the faulty logic applied by the NCTM:

- We want more students to become true mathematicians.
- True mathematicians love to figure things out for themselves and to spend a great deal of time working to solve strange, new and difficult problems.
- Therefore, as our *primary* way of teaching *all* students, we will let students figure things out for themselves and give them strange and difficult problems to work on.

This argument assumes that the means are the ends. But these means have not been shown to work well for most students, neither for lower-order nor higher-order problem types. Waiting for students to figure things out on their own does not usually result in them figuring things out on their own. Only some may succeed, not all.

To say that mathematics is about discoveries is sometimes true at all levels of schooling. It is particularly true at the graduate and post-graduate level. While Feinberg-McBrian can testify for herself, her family and her most successful students, for general recommendations we must concern ourselves with most students, paying particular attention to those who are at risk of failing ("naive learners").

How best to teach students to figure out things on their own is a question that is best answered by comparing how well different approaches achieve these results. When we say we need "research," we mean that it is important to look at actual student performance data in making these comparisons, rather than guessing or using examples limited to our own experience. Research has a vital role to play when we are considering the application of instructional methods on a broad scale, particularly with at-risk students.

According to Direct Instruction (DI) theory, those students who will eventually have the "faith" that inspires them to persist in solving complex new problems and to make new mathematical discoveries will be those students who have become academically competent in the tool skills of mathematics and have experienced abundant success in using those skills. The theory predicts that simply presenting difficult problems to students who have few or no mathematical tools will not develop the tools nor the confidence to solve the problems nearly as well as directly teaching the tool skills.

Feinberg-McBrian seems to agree that tool skills are important: "In giving complex problem-solving activities to disadvantaged children, the teacher must be particularly careful to pre-teach any component skills that the children would need in order to discover the solution, since the children have had no experience with this type of problem" (p 38). We cannot agree more. Tool skills must be taught if students are to experience success.

Feinberg-McBrian points out that the tests used to evaluate Project Follow Through did not "assess a child's ability to devise unique solutions to complex mathematical problems" (p 37). So, evidence supporting the assertion that DI in the tool skills of mathematics will lead to a better ability to solve novel mathematical problems is somewhat meager in the Follow Through data. But other experimental evidence indicates that direct teaching more effectively improves students' ability to discover than continually challenging students to discover solutions to mathematical problems as the NTCM asserts (Carambone & Holyoak, 1989; Craig, 1956; Kittell, 1957; Wittrock & Twelker, 1964). The evidence for the NCTM assertion consists primarily of anecdotes, such as those Feinberg-McBrian offers, about a few very talented individuals. An important question to ask is: What exactly were the components of success for these individuals? Did they simply have a unique response to challenge? Or did they perhaps have a more complete set of background skills that enabled them to respond successfully to the challenge?

It is fair to say that DI does not present tasks that are greatly challenging, and generally limits tasks to those closely aligned to what students have been taught. DI uses the limited amount of instructional time available to prepare students for greater challenges by providing them with solid mathematical tools that have wide and relevant application. This wide and relevant application continues to occur beyond the DI program. It is the culmination of the learning process that life provides.

We disagree with Feinberg-McBrian's view that DI has limited use. The DI approach, when used with more sophisticated learners, is not limited to "repetitive skills, such as memorizing the basic math facts" (p. 36). DI has been used successfully to teach a variety of higher-order thinking skills. (A 1992 Pro-Ed publication is devoted entirely to this topic, see ad in this issue of *Effective School Practices*.) The *Connecting Math Concepts* DI program teaches higher-order content such as geometry and problem solving. The *Reasoning and Writing* program teaches critical analysis of reasoning. The terms "sophisticated" and "naive" are relative. Every "learner" has already learned certain things, and every learner is "ready" for the next step. DI is an effective and highly efficient vehicle for carrying students to the next step, into unfamiliar domains.

There is no evidence that would contradict this approach. There is no evidence that providing students with mathematical tools will *disable* them, and hinder them from solving complex challenging problems. A recent comprehensive review of the experimentatl and descriptive research on creativity concludes that a relevant knowledge base is crucial for creativity and finds that spontaneity and sudden inspiration are largely overrated by society (Weisberg, 1986). Later issues of *Effective School Practices* will explore these, and related questions in greater depth.

Teaching for Understanding

DI programs such as *Connecting Math Concepts* also teach students to think of a concept in more than one way so that they truly understand it (something Feinberg-McBrian states cannot be taught by DI). For example, students learn that a fraction of a quantity is the fraction times that quantity:

> **e.g., 2/3 of the children are boys. There are 27 children. How many boys are there?**

$$B = \frac{2}{3} \times C$$

$$B = \frac{2}{3} \times 27$$

$$B = \frac{54}{3} = 18$$

There are 18 boys.

Students also learn that same problem may be expressed as a proportion:

$$\frac{\text{boys}}{\text{children}} \quad \frac{2}{3} \qquad = \frac{\square}{27}$$

$$\frac{\text{boys}}{\text{children}} \quad \frac{2}{3}\left(\frac{9}{9}\right) = \frac{\boxed{18}}{27} \qquad \boxed{\text{18 boys.}}$$

The issue is not whether to present one solution to a problem or to encourage alternatives. It is a question of timing. For naive learners, perhaps for most learners, to juxtapose alternatives during initial teaching will be confusing, and to encourage alternatives without sufficient pre-teaching favours a minority of students who are the most able, or who have the most advantaged home environment.

A second example of learning alternative ways to solve a problem also involves multiplication. For example, students learn that if a problem gives information about one item and asks about more than one of those items, the problem may be solved by multiplication, e.g., **There are 3 feet in 1 yard. How many feet are in 8 yards?** The problem tells about 1 yard and asks about 8 yards, so we multiply:

$$\begin{array}{r} 8 \\ \underline{\times 3} \\ 24 \end{array}$$

The answer is 24. Students also learn to set up an equation to express the relationship between units, e.g.,

$$\text{yards} = \frac{\text{feet}}{\text{feet per yard}}$$

The problem above may be expressed as:

$$8 \text{ yards} = \frac{\square \text{ feet}}{3 \text{ feet per yard}}$$

$$8 = \frac{\boxed{24}}{3}$$

So 8 yards is 24 feet. This same analysis is later applied to non-convertible units such as miles and hours:

$$\text{hours} = \frac{\text{miles}}{\text{miles per hour}}$$

Feinberg-McBrian states that DI cannot teach students how to tackle new problem types. However, the fraction analysis described above is included because of its generative properties, enabling students to solve a range of new problem types. What students have learned about measurement conversion can be applied to rate.

A similar approach is taken in the initial levels of the Reading Mastery program, providing students with a sounding-out strategy that can be applied to read new words that a student has never seen before. Generalization, or solving unfamiliar problems using what one has learned, is a fundamental aspect of Direct Instruction.

Issues of Scripted Instruction

Feinberg-McBrian describes her departure from the DI script as an "indirect approach." The teaching interaction recorded by Feinberg-McBrian as she taught doing. Scripts can orchestrate a carefully engineered sequence of learning tasks and teaching examples with carefully planned comments by the teacher. The timing of these comments must match the awareness students are gaining as they work items that are specifically engineered for this purpose. Scripts can empower more teachers to execute this orchestration successfully. Teachers who teach mathematics successfully without scripts do not need scripts; scripts are necessary only for teachers *who would benefit from such help.*

The use of scripts does not imply that a teacher should not be responsive to the students. Quite the contrary, teachers should be highly attuned and responsive to student performance as they teach. This is one of the most difficult things for people to understand about DI: It is possible to follow a very specific instructional procedure *and* be highly responsive to students. This is no different from a doctor following a highly specific surgical procedure, but responding to the patients needs competently and efficiently. Learning to be responsive while following a specific procedure sometimes requires rather extensive teacher training. A knowledgeable DI teacher would not simply continue through the script when students are having difficulty. Serious persistent errors would warrant the teacher reteaching an earlier component, just as Feinberg-McBrian did.

At the heart of DI is the notion of the teacher as a "responder to the needs of the student" as instruction proceeds. We have no doubt that Feinberg-McBrian is an excellent and responsive teacher. We hope that she continues to explore DI in its application to higher level mathematics and to more sophisticated learners.

Closing Thoughts

The ultimate resolution of the issues Carol Feinberg-McBrian raises depends on how we define the national educational problem that we are trying to solve. In the days after the Russians launched Sputnik, we saw our educational problem as the challenge to create an elite group of high-powered scientists and mathematicians. We needed inventors and Nobel prize winners to make us competitive in the space race. For that type of reform, the NCTM recommendations may have merit.

The educational problems facing us as we enter the 21st century are very different. Today, the educational challenge is to raise the performance level of the lowest one-third of our population. Approximately one-third of the population may be characterized as educational "have-nots." While many well-paying jobs have been available for this poorly-educated segment of the population up to this point in time, this will no longer be true in the 21st century. More and more jobs will require technical skills, and well-paying jobs for unskilled laborers will practically disappear.

The current impetus for educational reform came from the awareness of this new kind of problem. The NCTM recommendations seem inadequate when considering the needs of this lower one-third of the population. On the other hand, Direct Instruction has been proven effective, particularly with this population. Yet, as Feinberg-McBrian points out, there are those who would not attend to that research simply because it conflicts with their values. One must question those values. Whether Direct Instruction can result in the deeper understanding and higher-level thinking that so many educators value so highly, is a question that experimental research should answer. Whether Direct Instruction has negative side effects on other aspects of development is also a question that experimental research should answer. Translating values into action requires a research base. Our hope is only that educators will allow scientific evidence to inform our profession. To use the reigning educational dogma alone is not only insufficient, but unethical.

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Research-Based Guidelines for Selecting Mathematics Curriculum

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Why Another Set of Adoption Guidelines?

In the early 1900's, the challenge of integrating thinking and content area knowledge was formidable, even in the education of the elite. Teachers today face much greater challenges. "Although it is not new to include thinking, problem solving, and reasoning in *someone's* school curriculum, it is new to include it in *everyone's* curriculum"(Resnick, p. 7).

Additionally, regular classroom teachers face even greater challenges as a result of the Regular Education Initiative (REI), the movement that results in the placement of increasing numbers of special education students into regular classrooms.

The purpose of these adoption guidelines is to provide assistance for teachers, through the adoption process, in their quest to meet exceptionally difficult challenges with respect to their mathematics programs:

1. Effectively teach this extremely diverse group of students. Regular classrooms were quite diverse before REI, and have become more diverse since. Because of the multitude of ways students are referred for special education services, it has been estimated that up to 80% of American school children could qualify for special services in mathematics somewhere. That is to say, a teacher is likely to have several "regular" students in the room who are having difficulty with mathematics, while at another school, these same students would qualify for special services using a different scheme for referral.

2. Teach important mathematics. Rightfully, mathematics educators are calling for a much greater emphasis upon the mathematics that students are most likely to find useful in their lives.

These guidelines have been developed specifically as a means of addressing these two challenges effectively: teaching important mathematics to widely diverse learners. Consequently, they should be thought of as supplemental guidelines. They are modest guidelines, in the sense that they do not address the kinds of general criteria commonly listed in adoption guidelines: lack of sexual and other bias, attractiveness, and so on. However, there is little doubt that the instructional focus of these guidelines is becoming increasingly critical to teachers.

Figure 1 illustrates the nature of the new challenges. Traditionally, the curriculum in regular classrooms with no mainstreaming focused upon a relatively narrow "middle" section of students: all but the very lowest and the gifted. The lowest, "at-risk" students would eventually be retained, or would struggle along, or would be referred for special education or other specialized, basic services. Gifted students, similarly, might be referred to special programs for the gifted, or be accommodated by enrichment activities designed by teachers or provided as supplements to instructional materials.

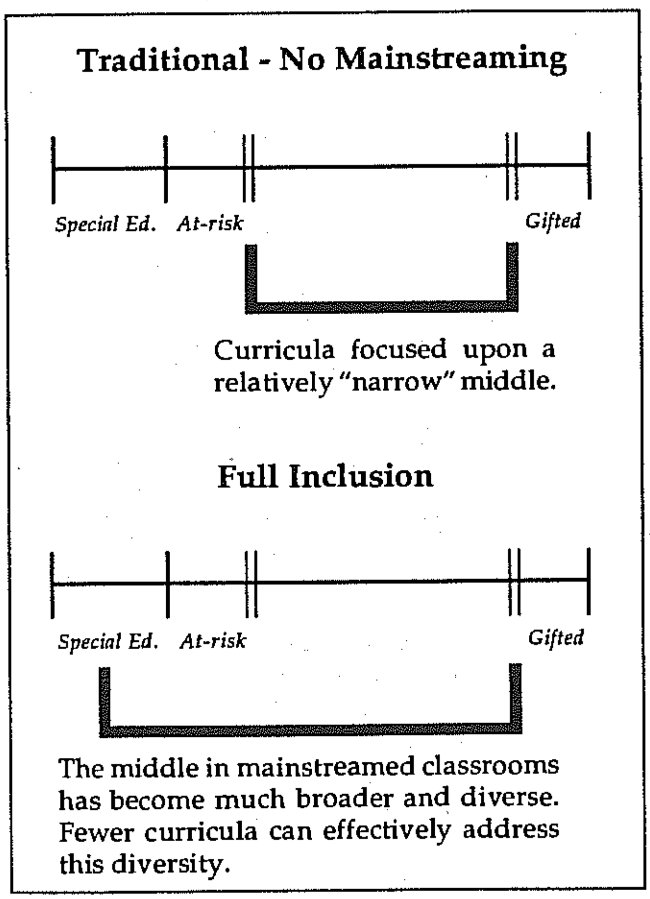


Figure 1. Curricula for Diverse Learners

With mainstreaming (and with "full inclusion" models especially), the diversity in a regular classroom can be staggering. Both the number and diversity of the "middle" students in such classrooms has increased. Imagine, for example, a classroom comprising twenty "regular" students and eight special education students. Imagine further that *all* students are mainstreamed. Of the eight special students, perhaps two have handicapping conditions such that they will require attention from a specialist in the classroom. Of the twenty regular students, a fourth or more may have difficulty with mathematics: they are behind the other fifteen students, and are at risk of eventual classification as special education students. One or two of those twenty regular students may be gifted. If the classroom teacher has primary responsibility for the fifteen regular students who are average or above, and a specialist has primary responsibility for the two most severely handicapped students, who has responsibility for the eleven "low" students? How many different mathematics lessons can one reasonably expect the teacher and/or the specialist to deliver each day? A traditional curriculum that targets the low-average to high range is pitifully inadequate for today's diverse regular classrooms.

Materials selected according to the criteria outlined in these guidelines can go a long way toward helping to make mainstreamed classrooms more manageable

Such difficulties, of course, cannot be entirely resolved through the judicious selection of instructional materials. However, materials selected according to the criteria outlined in these guidelines can go a long way toward helping to make mainstreamed classrooms more manageable. The few criteria focused upon in these guidelines are those shown by currently available research to promise the greatest acquisition of mathematics knowledge by the broadest range of diverse learners. An instructional program based upon these criteria cannot be expected to be the ideal curriculum for two groups of students within a diverse classroom: the most severely handicapped, and the very highest performing students.

Returning to the classroom scenario above, the guidelines are intended to help adoption committees select materials that will be effective with the six "non-severe" special education students, the five low regular students, and perhaps all but the five highest of the remaining fifteen regular students. The regular teacher could teach mathematics to those twenty-one students using such materials, while the specialist continues to have primary responsibility for the severely handicapped students. And what of the five highest students? As you will see, these guidelines are not likely to restrain the mathematics growth of higher-performing students. As a consequence, those five could participate with the other twenty-one in the regular mathematics lesson, so long as that work is supplemented (by either the specialist or the teacher) with more challenging material.

The focus of these guidelines, then, is upon all but the very lowest students (who require the services of specialists in any circumstance) and the very highest (who are the most able to work independently, and to learn well from indirect, discovery-oriented activities).

Although circumstances vary considerably, and scenarios like the one above are not perfect solutions to complex problems, accommodating the greatest possible diversity through the low to high-average range promises greater manageability in classrooms with mainstreamed disabled learners.

The Organization and Use of These Guidelines

There are six individual guidelines presented herein. Each is summarized on one of six single-page charts. Essentially, those charts *are* the adoption guidelines. That is, we cannot realistically expect every member of an adoption committee to read the document you are reading now. Rather, we anticipate that perhaps one member of a committee will read this document: a committee chairperson, or someone assigned especially to investigate the needs of mainstreamed learners. In that sense, this document is a backup to the six guideline charts. The charts, and this document as well, are backed up further by a technical report (Dixon, 1992).

As a practical matter, each member of an adoption committee needs only the six guideline charts in order to evaluate the extent to which various instructional materials accommodate diverse learners. Although guidelines may be applied to materials in any order, we suggest:

1. Big Ideas
2. Conspicuous Strategies
3. Mediated Scaffolding
4. Strategic Integration
5. Primed Background Knowledge
6. Review

Generally, this is the order of importance. For example, #1 - Big Ideas, refers to teaching important mathematical concepts. Other guidelines, such as review, are not likely to impact much on understanding if a program focuses attention on unimportant, or even trivial concepts.

The Guidelines

Big Ideas

In order for students to solve interesting, complex, realistic problems, they must acquire knowledge of the more important, key mathematical concepts: concepts recently referred to in mathematics literature as "Big Ideas." Big ideas within a content area are those concepts, principles, or heuristics that facilitate the greatest amount of knowledge acquisition and understanding across the rest of that content area. That is, big ideas make it possible for students to learn the most and learn it well, as efficiently as possible. Consider, for example, the following problem:

> At lunch, each student can choose a carton of white or chocolate milk. Each fifth-grade class is to estimate how many cartons of chocolate and white milk should be ordered for the entire school.

In addition to computational ability, for students to work such problems successfully, they must fully understand the big mathematical ideas of proportions and estimation, concepts that recur frequently in a broad range of real mathematical problems.

An all too common alternative to big ideas is broad *coverage* of or *exposure* to numerous objectives. Neither the term coverage nor exposure implies the kind of learning with understanding to which educators generally aspire for regular classroom and mainstreamed learners alike.

Several "small" mathematical ideas can often be best understood in relationship to a larger, "umbrella concept." For instance, if the geometric concept of area is taught to a high level of understanding, then students need not learn formulas by rote for calculating area. In turn, the understanding of area provides the underpinnings of another important mathematical concept: volume, as a function of base (area) times height. Approached this way, the seven traditional formulas for computing volume can be reduced to a single formula (base times height) with two slight variations.

Finally, even something as "simple" as math facts can be taught in terms of larger mathematical concepts, rather than as a large set of discrete and unrelated facts to be recalled by rote. For instance, each addition fact bears an "adding on" relationship to some other fact: 4 + 4 = 8, and 4 + 5 is one more than 4 + 4, so the sum is also one more, 9. A more sophisticated approach to facts is to capitalize upon the inherent relationships among addition and subtraction facts alike. For example, all four of these problems: 3 + 4, 4 + 3, 7 - 4, and 7 - 3 are all part of a single "family" of relationships. Similarly, "larger" problems also bear family relationships to one another: 1003 - 989, 989 + 14, etc. Such interrelationships can be mapped:

$$\xrightarrow{4 \qquad 3} 7$$

$$\xrightarrow{989 \qquad 14} 1003$$

The mapping strategy can also be expanded beyond facts to solve verbal math problems.

Evaluating Big Ideas. Although authorities in mathematics education are increasingly emphasizing the importance of teaching big ideas, they are yet to develop a definitive list of important mathematical concepts. If there were such a list, adoption committees could simply check off program concepts against a list. Still, several concepts seem to be generally accepted as crucial, big ideas: place value; the addition, subtraction, multiplication, and division of whole numbers; fractions, including ratios and proportions; estimation and approximation; probability; volume and area; and all of these concepts incorporated into verbal problem-solving.

In an instructional program that emphasizes big ideas, a given idea is likely to appear repeatedly, in different contexts. For instance, the "identity principle" in mathematics says that the value of a number does not change when multiplied by one, *in any form* (such as 23/23, 4ay/4ay, etc.). Such an idea is big (key, important) because it facilitates understanding of so many aspects of mathematics, ranging across fractions, ratios, and algebra. Thus, one step in evaluating an instructional program for big ideas is to search for a major mathematical concept that is applied in a variety of contexts.

Another step in evaluating big ideas is effective, yet relatively simple. Significantly more time will be allocated to big ideas than to minor ideas. At a given level of an instructional program, a single big idea and a relatively small idea can be compared upon the basis of the time allocated to teaching (or covering or exposing) each. If, for example, all outcomes in a program are pursued in units or chapters of equal length, that program *does not* differentiate big ideas from others. In contrast, if certain ideas occur repeatedly in a program and more time is allocated to those ideas than to several others, then big ideas are probably being accommodated.

The highest performing students are those most likely to, eventually, infer useful strategies from their experience. Explicitly taught strategies, in effect, let the majority of students in on the "secrets" to success.

Conspicuous Strategies

Someone who easily and fluently solves new mathematical problems uses some kind of "strategic approach" to each new problem-solving situation: some kind of expert strategy. In many cases, experts cannot even clearly articulate the details of their strategies. The purpose of strategy instruction is to clearly present learners with strategies like those that experts use. The highest performing students are those most likely to, eventually, infer useful strategies from their experience. Explicitly taught strategies, in effect, let the majority of students in on the "secrets" to success.

A strategy, then, is a somewhat general set of steps students follow in solving problems. Strategies may be so narrow that they result in rote-like performance on a very limited set of problems. In contrast, a strategy can be so broad that it doesn't work for the majority of students, the majority of time. Simply "drawing a picture," for example, can in fact help some students solve some problems, but is too broad to qualify as a reliable strategy for most students.

"Medium" strategies are the most likely to benefit students. Consider this problem:

It takes the attendance office two minutes to process three tardy students. How long will it take them to process eleven tardy students?

A "medium" strategy is to first map the "units" in such a problem:

$$\frac{\text{minutes}}{\text{tardy students}}$$

Next, write the known quantities with the units:

$$\frac{\text{minutes}}{\text{tardy students}} \qquad \frac{2}{3} \qquad 11$$

Finally, map the missing quantity:

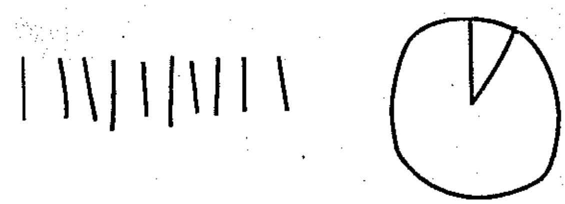
$$\frac{\text{minutes}}{\text{tardy students}} \qquad \frac{2}{3} = \frac{\square}{11}$$

All that remains is the computation necessary to solve this correctly mapped problem.

Evaluating Strategies. First, determine whether a strategy is taught explicitly at all. Only a limited number of students are likely to infer effective strategies, and the process of inferring strategies is very time-consuming (inefficient) in any case.

Next, make a preliminary determination of how narrow or broad strategies appear to be. Can the strategy be applied to a large number of problems? Can it be applied reliably?

Determining the "reliability" of a strategy can be a challenging and engaging activity. A good technique is to role-play a student, one that is on the lower end of the targeted student population, and perhaps one that is a bit contrary. In that role, the objective is to attempt to "break the strategy" by *identifying any instance in which one can follow the strategy, but still not solve the problem.* As a simple example, a student could easily "draw a picture or map" of the tardy student problem above, but without indicating the relationships necessary to solve the problem. Here is one such student drawing:



Playing the role of the student can take a little practice. It requires one to focus strictly upon the information given in strategies and problems and not use one's own knowledge. An approach that can be effective is for someone on the adoption committee to locate problem types in the upper levels of the material under consideration that the committee person has genuinely had a difficult time understanding for himself or herself in the past. If the strategy does not facilitate understanding for such an educated adult, it certainly is unlikely to facilitate understanding for students. It is usually safe to generalize the findings from evaluating the upper levels of materials to the lower levels.

Mediated Scaffolding

British educator A. J. Romiszowski has characterized traditional mathematics instruction as: "I'll work two on the board, then you do the rest." The "I'll work two" part of that approach can be thought of as a model, and the "you do the rest" is considered immediate testing. It has been said that the problem with learning from experience is that the lessons come too late. The same could be said of this traditional model of instruction. After "doing the rest," students might receive feedback—ranging from right/wrong to an explanation of how to do missed problems—and possibly a grade. The feedback is too late, and the grade, too early.

Scaffolding is a means by which students receive support in various forms along the path to full understanding and "doing the rest" successfully. An analogy helps illustrate the potential power of scaffolding in mathematics instruction. Imagine trying to teach a physically disabled youngster or a very young child how to slide down a playground slide. We might begin by carrying the child up the steps and holding her in our lap as we slide. That first phase of teaching illustrates a great deal of support—scaffolding.

After proceeding this way a few times, we might remove a bit of the scaffolding by, perhaps, allowing the child to take one or two steps of the ladder with minimal help, or by placing the child low on the slide and allowing her to slide on her own a very short distance. And so on.

All along, we would attempt to remove more bits of scaffolding, but in no instance would we abruptly remove all the scaffolding and, in essence say, "you do the rest." Obviously, doing so could result in serious physical injury. Analogously, removing scaffolding prematurely—or worse, never providing any—can result in serious intellectual injury for many students. Such injury is less obvious than physical injury, but just as real, just as likely, and perhaps over time, more impairing. After all, physical injuries heal. Many mathematical injuries are extremely difficult to rehabilitate.

Teachers and peers can provide scaffolding, independently of instructional materials. However, materials can and should provide scaffolded sequences of tasks for students to perform. Highly scaffolded tasks may be quite contrived and "look strange," just as carrying a child in the early steps of "sliding" does not look much like "real sliding." The crucial thing is that tasks clearly and systematically provide gradually reduced scaffolding so that ultimately, students solve mathematical problems effortlessly, and with few injuries along the way.

Evaluating Scaffolding. Select an important strategy from a grade level, preferably a strategy on a concept already identified as a big idea. Focus examination on all the tasks associated with the selected concept, from the very beginning, to the point at which students are expected to apply their knowledge independently.

The first tasks should be designed to ensure that even relatively low students can perform them successfully. Gradual—perhaps even subtle—changes in tasks should be discernible, changes that reflect a shift to greater student understanding and independence.

Strategic Integration

The principal benefit to all learners of integrating knowledge is that it enhances the likelihood that students will learn *when to apply* their knowledge. There are many concepts in mathematics that are similar to others, yet significantly different. For instance, from the point of view of someone just learning fractions, an addition problem and a multiplication problem appear to be very similar to one another, yet very different knowledge is required to solve each:

$3/4 + 4/5 = ?$ $\qquad$ $3/4 \times 4/5 = ?$

The benefit of integrating is probably most apparent when compared with the difficulties students encounter with "non-integrated" knowledge. They may, for example, simply add the numerators and denominators of addition fraction problems such as the one above, in spite of the fact that when adding fractions was first being taught, students seemed to "get it." Perhaps they had little trouble with adding fractions initially, when addends had like denominators. Perhaps they subsequently caught on to adding fractions with unlike denominators. But after teaching multiplication of fractions, without integration, confusion developed: Which problem type requires like denominators? When does one operate on both the numerator and the denominator?

The predictable outcome of not integrating knowledge is an abundance of stubborn misconceptions: using area formulas to compute volume, subtracting a big number from a small number in whole number subtraction, difficulties with fractions with values greater than one, and so on. Although such misconceptions are related to the quality of initial teaching (big ideas, strategies), even topics that are well understood initially can result in confusion without integration.

The primary means of integrating knowledge is quite simple, perhaps elusively simple: just do it! That is, the developers of instructional materials can considerably advance the goals of fully integrated knowledge by "putting things together." In the "real world of mathematics," we are constantly called upon to pull some specific knowledge from our entire base of mathematics knowledge and use that specific knowledge appropriately. Instructional materials should anticipate that, in addition to possessing knowledge, students need to learn *when* to use components of that knowledge.

Evaluating Integration. Select any common mathematics misconception which committee members have observed in students. Identify specifically what gets confused with what in that misconception. For example, if students add the numerators and denominators of fractions, they are probably confusing addition with multiplication. Examine the materials under consideration to determine the extent to which the potentially confusing concepts are mixed with one another.

Initial teaching examples can also be used to facilitate integrated knowledge. If all the examples students work with as they initially learn fractions have a value of less than one, then it is quite predictable that many students will draw the reasonable—but wrong—conclusion that all fractions are "small pieces of things," with values of less than one. Look at initial sets of teaching examples to determine whether some inadvertent misconceptions are actually implied by the examples used.

New problem types can be related to familiar problems taught earlier. Students can be shown specifically why Strategy "A" is not adequate for solving problems in "B" situations. Look for examples such as: "This looks like a problem that can be solved with _______ procedure, but that won't work. Here's why: ___________." An example of one concept or strategy is always a "negative example" of another. Full understanding requires that student know not only what something is, but also, what it is not.

Primed Background Knowledge

It nearly goes without saying that learning most mathematics, and especially most important mathematics, is dependent upon some knowledge the learner already possesses. This fact is generally accommodated in most instructional materials, but not always effectively.

Three different practices are of limited value. The first is teaching prerequisite knowledge through "coverage" or "exposure." If prerequisite material was just covered, or students were otherwise exposed to it, it is not likely to have been learned well enough to facilitate the learning of new, complex knowledge.

Even when prerequisite knowledge is taught thoroughly, the timing of that instruction in relation to the new knowledge can be crucial. The second and third practices that are inadequate relate to timing. On the one extreme, the prerequisite knowledge may have been taught far in advance of the instruction on the new knowledge. In that case, many students are likely to lose their facility with the prerequisite knowledge, if not to forget it altogether. At the other extreme, prerequisite knowledge is sometimes taught in too close proximity to the new knowledge, often on the same day, during the same instructional lesson. In this case, students have no time to truly learn, assimilate, and understand the prerequisite knowledge before the new knowledge is introduced.

Ideally, prerequisite knowledge is introduced (or reviewed) at a given point, and used for a period of a few days or weeks before the introduction of the new, more complex knowledge. Such a schedule means that students have the opportunity to develop sufficient fluency for using the prerequisite knowledge, without the opportunity of having that fluency diminish or disappear as a result of disuse.

Evaluating Prior Knowledge. To evaluate material based upon this guideline, it is necessary to locate one or more complex topics in a given level of the instructional program under consideration. Next, that topic must be analyzed to determine what prerequisite knowledge is assumed for learners. First, determine whether the prerequisite knowledge has been taught (or reviewed) thoroughly (rather than simply covered or "taught for exposure").

Next, note the time interval between the instruction on the prerequisite knowledge and the new knowledge. A "medium" time interval (of a few days or weeks of nearly continuous, if light, practice) is desirable. Neither long intervals without intervening practice nor extremely short intervals are likely to be effective.

Review

The term "review" can be an emotive one in education, conjuring up images of endless (and, perhaps, mindless) drill and practice. Yet research strongly supports certain review practices as significantly effective. We include review as the last guideline because in many ways, effective review is dependent upon the extent to which other guidelines are implemented in instructional materials. It can be said that one gets out of review what one puts into it. That is, the quality of instruction—principally in terms of big ideas and strategies—influences the value of review. Regardless of how "small ideas" or marginally significant material is reviewed, the ideas remain small, the material marginally significant.

The following are requirements for effective review:

1. **Sufficient.** Is there enough review to achieve the goals of fluency and understanding?
2. **Distributed.** Given a fixed number of review opportunities, that number will enhance learning better if it is distributed over time than if it is massed. Specifically, distributed review contributes to long-term retention and automaticity of knowledge.
3. **Cumulative.** This requirement is tied closely to the integration guideline. The notion of cumulative review means that material taught accumulates in review. After A and B are taught, for example, A and B are reviewed together.
4. **Varied.** With relatively few exceptions, the specific items that are reviewed should not be the same as the items used earlier in instruction. The reason for this is that varied items promote generalization and transference. However, items should not be so varied that they actually represent new knowledge.

Evaluating Review. It is impossible to give a simple formula for determining how much review is sufficient. Instructional materials, however, should err on the side of "too much" review. When students need less review than a program provides, teachers can simply cut back the assignment of review work. In contrast, a great deal of teacher time and effort would be required to create additional review if the amount provided in a program is insufficient.

When examining a program for the distribution of review, the general amount of review can be noted, too. Evaluate the distribution of review by creating a summary of the review for a major concept taught in the program under consideration. Only tasks that students complete independently (including test tasks) should be considered "review." The results of such an evaluation can be represented in many ways. Here is one example of visually representing well-distributed review:

	Lesson Number								
	23	24	25	26	27	28	29	30	31
Number of Review Tasks	7	5		4		4			6

Determine whether the review in a program is cumulative. Select concepts that are frequently confused by students (multiplying and adding fractions, decimal versus whole number place value, subtracting with regrouping and adding with regrouping, etc.). Are the confused concepts reviewed together?

When review is cumulative, some form of scaffolding should be provided initially. For example, if students are reviewing the addition and multiplication of fractions together for the first time, the task should prompt the students to carefully note the operation in each problem before beginning computation.

Finally, determine whether review items vary appropriately. Assume, for example, that algorithms have been taught for subtracting whole numbers with no renaming required, and problems requiring renaming but with no zeros in the minuend. Both types of problems can, literally, be infinitely varied by varying the individual digits from 1 to 9, and by varying the number of digits in both the minuend and the subtrahend. However, problems including zeros in the minuend would not be a variation of the algorithms taught, but rather, they would represent new knowledge.

Possibly, the most difficult area of mathematics to evaluate for appropriate variation is problem solving. Frequently, instructional programs include many "review" problems that cannot actually be solved by following strategies previously taught. There are few tidy listing of problem types in mathematics; basically, a problem type is defined by the strategy taught to solve that type. Thus, evaluators should actually work a variety of review problems, being careful to apply only strategies taught in the program under consideration. If other, untaught knowledge is required to solve some "review" problems, then the variation of review is too great.

Summary of Guidelines and Impact on Diverse Learners

In developing these guidelines, it has been crucial to not only identify principles of effective instruction for lower-performing students, but to focus upon those principles that are effective with many other learners in mainstreamed regular classrooms as well. We stated in the beginning that instructional materials incorporating these guidelines are likely to be appropriate for all but the lowest and highest students in a highly diverse regular classroom. Specifically, these guidelines impact diverse learners in the following ways:

1. **Big Ideas.** Because the notion of "big ideas" is roughly comparable to important ideas, knowledge, and concepts, this principle is appropriate for all students, without modification for different ability levels. The principle of big ideas is learner independent: mathematical concepts that are important for understanding are important for everyone.
2. **Conspicuous Strategies.** All students can benefit from explicitly taught strategies. That, however, is not to say that every student requires explicit strategies as much as any other. Gifted students, for example, can benefit from explicitly taught strategies, but require such explicit instruction to a lesser extent that their lower-performing peers.
3. **Mediated Scaffolding.** The amount of scaffolding for any given strategy appears to vary as a function of ability. Lower students need more scaffolding; higher students need less. Instructional programs should provide scaffolding that is adequate for the lower-performing students, giving teachers the option of cutting back on scaffolding without needing to develop more than a program provides.
4. **Strategic Integration.** Knowing when to use knowledge is important to understanding for all learners.
5. **Primed Background Knowledge.** In theory, all students need the knowledge that is prerequisite to acquiring new, complex knowledge. In practice, however, it is the lower-performing students who are most likely to display deficits in prior knowledge. Either they never learned such knowledge, or did not learn it thoroughly, or learned it fairly well, but forgot it.
6. **Review.** The principles of effective and efficient review are generally most crucial for lower-performing students. Specifically, higher-performing students are likely to require less review. However, integrated (cumulative) review is important to understanding for all students, and varied review is important to transfer and generalization, regardless of ability level. The consequences of inappropriately varied review are most likely to cause difficulties for the lower-performing students. As is the case with scaffolding, instructional materials should provide review sufficient for the lower-performing students.

Note: Research references have been largely omitted from this report for ease in reading. A complete technical report supporting these conclusions can be obtained from the National Center to Improve the Tools of Educators, 805 Lincoln, Eugene, OR 97402. Ph: 1-503-683-7543. Ask for the technical report entitled: "Mathematics Guidelines for Diverse Learners."

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Considerations For Adopting Mathematics Instructional Materials
Big Ideas

National Center to Improve the Tools of Educators • Phone (503) 683-7543
Funded by the US Office of Special Education Programs

In order to promote deep understanding and problem-solving, students should be taught major, important mathematics concepts. That knowledge will make learning "subordinate" concepts easier and more meaningful. Big ideas represent central ideas within a discipline. They have rich explanatory and predictive power. And perhaps foremost, Big Ideas apply to many common, everyday contexts and situations.

Example: Volume is frequently taught as seven similar and confusing formulas that many students attempt to simply memorize. An alternative is to teach the fundamental concept of volume as a function of bases times height. When students have mastered the concept of "base," they can calculate any volume with two minor variations on the basic formula, area of the base times height: B x h.

Rectangular prisms, wedges, cylinders:	B x h = v
Rectangular pyramids, triangular pyramids, cones	B x 1/3 h = v
Spheres:	B x 2/3 h = v .

Rationale: Topics in mathematics are often treated as unrelated. Moreover, there has been a proliferation of objectives or topics in published materials over the past several years. As a consequence, many topics are just "covered" and relatively minor topics get as much attention as very important topics: "big ideas" in mathematics. In order to understand mathematics, students should thoroughly *learn* (as opposed to "cover") the most important mathematics concepts.

Implications for Materials Evaluation. Big Ideas can be incorporated into instructional materials in a variety of ways, implying that materials can be examined for different manners in which big ideas may be accommodated.

1. Significantly more instructional time is allocated to big ideas than is allocated to other topics. For instance, if topics are typically addressed in weekly units, big ideas might be taught as sections of two or more units. This is a "quantitative" judgment: Big Ideas are the most important ideas, so more time (and/or space) is allocated to them.

2. Topics can be analyzed for "cross topic" big ideas, important mathematical relationships that apply to a wide range of traditional mathematical topics. For example, number families (e.g., $\xrightarrow{4 \quad 2} 6$) represent the relation between addition and subtraction. For instance, $\xrightarrow{4 \quad \square} 6$ calls for subtraction, 6 - 4 = □). The use of number families provides a single basis for a higher order organization for facts, a mapping strategy for problem solving, a method for analyzing data tables, and a foundation for simple algebraic relationships, such as 4 + □ = 6, so 6 - 4 = □.

This is more of a "qualitative" judgment that requires evaluators first to identify those mathematics ideas they believe to be most central, then to examine all those topics to which each important idea is most likely to apply. For example, if the "identity principle" is considered a crucial mathematics concept, then one might reasonably expect to see it used conspicuously in teaching division of fractions for deep understanding.

Considerations For Adopting Mathematics Instructional Materials

Conspicuous Strategies

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A strategy is a series of steps that students follow to achieve some goal. Such steps are an approximation of the steps experts follow covertly (and, perhaps, unconsciously) while working toward similar goals. In instruction, such steps are initially made overt and explicit for students. Eventually, as students master a strategy, the steps become more covert, as for experts. Strategies should be specific, but "intermediate in generality," neither too narrow nor too broad.

Example: A strategy for learning math facts efficiently is to use number families, which look like this:

$\overset{4 \quad 5}{\xrightarrow{\hspace{2cm}}} 9$ Four facts are represented in this family: 4+5, 5+4, 9-5, 9-4. Students can use the same basic number family strategy to solve more difficult-sounding "algebraic" number puzzle problems. The strategy is to determine whether a "small" number is missing (requiring subtraction) or a "large" number (requiring addition). In

$\overset{31 \quad \square}{\xrightarrow{\hspace{2cm}}} 46$, a small number is missing, so students subtract. The same basic strategy can be applied further to verbal problem-solving: *Juan had 16 more stamps than Frank had. Juan had 28 stamps. How many did Frank have?* Here is a map of this problem:

$\overset{16 \quad \text{Frank}}{\xrightarrow{\hspace{2cm}}} \text{Juan } 28$

Students create such maps by using specific steps. First, draw a number family arrow. Next, determine the large and small numbers that are given and place them on the map. Determine the operation by seeing whether the missing number is large or small. Complete the computation.

Rationale: The fundamental rationale for good strategy instruction is that it helps to ensure that all students "get it."

Implications for Materials Evaluation: Depending upon the currently popular *Zeit Geist* in education, instructional programs might not utilize overt strategies at all. Overt strategies provide an option to other, more covert and discovery oriented activities, as a means of accommodating those learners for whom discovery is ineffective or inefficient or both. Therefore, materials intended for use by learners with a broad diversity should provide conspicuous, explicit, overt strategies, as least as an easily implemented option.

Once evaluators have identified conspicuous strategies in materials, the next step is to determine whether those strategies are too broad or too narrow.

Conspicuous strategies, however, are not all created equal. A narrow "strategy" is rote-like and reflects little understanding: *invert and multiply.* A broad "strategy" only works sometimes for some students. For instance, the strategy of "drawing a picture" to solve verbal problems is insufficiently specific to consistently allow most students to solve many kinds of problems.

Finally, strategies that make sense to people who already know the objective of the strategy may not be entirely clear for those who don't: students. Evaluators should take care to examine strategies from the point of view of those who truly don't already know them.

Considerations For Adopting Mathematics Instructional Materials

Mediated Scaffolding

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Imagine that you wanted to help a toddler play on a slide. It is not likely that you would just show that student how to climb the ladder and slide down, and then say, "You're on your own now." More likely, you would help that student as much as necessary, gradually withdrawing your help as the child became more confident and competent. Scaffolding is that same type of assistance given to students in academic areas between the introduction of new knowledge and the eventual self-directed application of that knowledge.

Example: Difficult and complex strategies can be scaffolded with a series of tasks that gradually result in self-directed application. The following three tasks illustrate scaffolded steps for adding (or subtracting) fractions with unlike denominators:

Task 1

$$\frac{1}{4} = \frac{\quad}{12}$$
$$\frac{2}{3} = \frac{\quad}{12}$$
$$\overline{\qquad\qquad\quad \frac{\quad}{12}}$$

Task 2

$$\frac{1}{3} = \frac{\quad}{\quad}$$
$$+\ \frac{4}{5} = \frac{\quad}{\quad}$$
$$\overline{\qquad\qquad\quad \frac{\quad}{\quad}}$$

Task 3

$$\frac{3}{7}$$
$$+\ \frac{5}{3}$$
$$\overline{\qquad\qquad}$$

Rationale: Many children--if not most--simply will not learn and understand a complex strategy *without* some form of scaffolding. Some scaffolding can be in written form, as part of mathematics instructional materials, illustrated in the tasks above. Some scaffolding can be provided directly by teachers. For instance, instead of the written scaffolding in Task 2 above, a teacher could present Task 3 with the wording: "Remember to rewrite the fractions so that they have the same denominator. Finally, some can be provided by other students, through cooperative learning activities. However, an instructional program should facilitate scaffolding by providing sequences of tasks like those above.

Implications for Materials Evaluation: Some materials supply supplemental scaffolded tasks as part of a materials package. Such tasks would be appropriate for any student whose performance on independent tasks indicates that more work is needed before successful independent performance can be obtained.

A scaffolded sequence of tasks for every topic (or every important topic) taught in a program is the most effective means of giving students support. Task #3 above is a typical outcome task in many programs. Each of the previous, scaffolded tasks derives from that outcome task. Evaluators should identify all the tasks associated with any given Big Idea, then examine those tasks to determine whether they are scaffolded: i.e, they generally move from simpler and more contrived to more complex and natural.

Considerations For Adopting Mathematics Instructional Materials

Strategic Integration

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Integrated knowledge is the opposite of segmented knowledge. Knowledge is segmented when a given topic is taught, presumed learned, and then "dropped"; and a new topic is taught in the same fashion. One method of integrating knowledge is to combine different topics during review (See Review guideline). However, integration is also achieved through example selection during initial instruction and scaffolded instruction.

Example: Many common student errors occur as a result of a program's failure to integrate knowledge. For instance, a program might introduce these initial teaching examples for greatest common factor:

4/12 9/3 8/16 9/27 3/12 10/5

From this set, some students are likely to make the completely reasonable--but incorrect--assumption that the greatest common factor in a fraction is always one of the numbers in that fraction. An initial set of examples that preempts such confusion is: 12/30 4/12 12/18 9/27 6/8 9/12 16/8. In only three of these fractions is one number the largest common factor.

Other examples of knowledge that should be integrated:

- Subtraction problems with and without renaming
- Division with one-digit and two-digit divisors (and more as they are taught)
- Addition/subtraction and multiplication/division of fractions

Probably the most important type of integration is with various types of verbal problems.

Rationale: The principal importance of knowledge integration is that it promotes *understanding*. Students need to learn not only the "what" and "how" of mathematics, but the "when" as well. Many students, for instance, add the numerators and denominators of fractions addition problems. They haven't learned *when* they can operate on both clements of a fraction and when they cannot.

Implications for Materials Evaluation: One way in which integration is incorporated into materials is through cumulative review activities, in which all topics taught to-date are included. Such activities can be supplements to existing material, but are probably more effective as a built-in feature of materials. Short of cumulatively reviewing *all* topics taught, integration review should at least include a mixture of commonly confused topics.

A common frustration of school administrators, teachers, and others is student performance on standardized tests. Such tests can be characterized as large-scale cumulative reviews. That is, students must discriminate among all the items tested to perform successfully, which is also the way students use mathematics knowledge in the real world. Although there are valid criticisms of such tests, it seems probable that students would uniformly perform better on them if they didn't have to wait for a standardized test for the opportunity to integrate their knowledge.

Evaluators should examine materials to determine the extent to which they integrate important, potentially confusing topics (such as adding and multiplying fractions, identified above under "Examples").

Considerations For Adopting Mathematics Instructional Materials

Primed Background Knowledge

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Nearly every mathematics topic is taught with the assumption that students will build their new knowledge upon a base of existing, well-established prior knowledge. Instructional material accommodates this rather intuitive fact *in some way*, but frequently, required component knowledge is not *well-established*. An inadequate accommodation of prior knowledge may be a major cause of student difficulty with mathematics.

Example: There are common *inadequate* ways of accommodating prior, component knowledge: (1) simply "covering" requisite prior knowledge, not teaching it to a high level of mastery (see Big Ideas guideline), (2) teaching component knowledge adequately, but with a substantial gap between that instruction and the instruction on the new knowledge, and (3) introducing requisite component knowledge almost simultaneously with the new knowledge for which it is required. (See Rationale, below.) An *appropriate* schedule for teaching new knowledge and its components might look like this:

Week 1	Week 2	Week 3	Week 4	Week 5
Introduce Conspicuous Strategy for Component	"Heavy" practice on component	Begin "light" component practice	Continue light practice	Introduce the new knowledge
Scaffold Component				
Lowest Common Multiple	....	....	. . . .	Adding & subtracting fractions with unlike denominators

Rationale: Students must have *well-established* knowledge of lowest common multiples (LCM) as a prerequisite to adding and subtracting fractions with unlike denominators. If LCM is covered, but not taught thoroughly, then many students are likely to have difficulties adding and subtracting fractions. If LCM is taught thoroughly, but then "dropped" for several lessons before adding and subtracting fractions is introduced, many students will have lost their "automatic" facility with LCM. And finally, if LCM is introduced just prior to adding and subtracting fractions, there is no time for students to develop *well-established* knowledge of LCM.

Implications for Materials Evaluation: Evaluators should first determine what accommodations for prior knowledge are made in a program. One major accommodation of prior knowledge is a placement tool. The other is the sequencing of topics in materials. Determine whether topics are ordered so that requisite knowledge is taught prior to the introduction of new knowledge, without either too large a gap between requisite and new knowledge or no gap at all. Something taught "last year" or even "last month" is too large a gap, and prior knowledge introduced on the same day as the new knowledge is too narrow a gap.

Considerations For Adopting Mathematics Instructional Materials

Judicious Review

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Review should be (1) sufficient, (2) distributed, (3) cumulative, and (4) varied.

Example: Here is an appropriate review schedule for a topic that has been introduced and has gone through a period of scaffolded instruction that ends on Lesson 22. (See scaffolding guideline). At first, review occurs in every other lesson.

Lesson	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57
	R		R		R		R			R			R					R					R						R				Etc.			

Because the review in the example is *distributed*, the quantity of review *per lesson* does not have to be great: perhaps only two or three review opportunities per lesson. The distribution in the example in called an "increasing ratio" distribution--the number of lessons between reviews gradually increases.

Examples should be variations of what was taught. For instance, if students are taught a strategy for solving verbal problems in which the verb indicates the operation required, then *different* problems of that same type should be reviewed. The example below might be the first using the verb *gave*.

Dan had twenty marbles. He *gave* seven to Jill. How many does he have now?

Eventually, examples of one type should be reviewed with similar examples of different types (cumulative review). Problems in which the verb suggests an *opposite* operation, for instance, should be reviewed with those in which the verb suggests the operation.

Dan had twenty marbles. Jill *gave* him seven. How many does he have now?

Rationale: The *distribution* of practice (1) promotes retention and automaticity, and (2) helps promote transfer by requiring application of the content at different times and in different settings. *Variation* promotes transfer, but variation that is too far from what was taught does not; variation that is too great represents new learning, not review. Making review *cumulative* (1) helps prevent confusion, (2) promotes learn when to use knowledge, and (3) promotes the integration of knowledge. (See Integration Guideline).

Implications for Materials Evaluation. Evaluators can examine materials for evidence that any of the following four characteristics of effective review are present.

1. Sufficient. This judgment is subjective, but generally, more review is better than less, particularly for diverse learners, and with respect to Big Ideas for all students.
2. Distributed. Review can be distributed relatively easily by attending to some sort of distributed review schedule when creating "more review." A more sophisticated version of distributed review is an "increasing ratio" review schedule, such as that illustrated above.
3. Cumulative. (See Integration Guideline.)
4. *Appropriately* Varied. With a few unavoidable exceptions, such as basic facts, the same problems should not appear over and over again in review. (That would promote rote learning.) The more common problem, however is that problems vary *too much* : some review problems are of a different type than those taught. A strategy for subtracting without regrouping, for example, is not likely to work for problems that do require regrouping.

Research Brief: A Field Validation of *Connecting Math Concepts*

George Brent and Nicholas DiObilda, (July/August, 1993). Curriculum Alignment Versus Direct Instruction: Effects on Stable and Mobile Urban Children. *The Journal of Educational Research*, Volume 86, No. 6, pp. 333-338.

The City of Camden, New Jersey, received a Follow Through grant to implement the Direct Instruction model (DI) in one of its elementary schools. Approximately half of Camden's families receive welfare and about 60% of Camden's children live in poverty, the highest percentage in the nation (Kozol, 1991). Students beginning first grade in two elementary schools (K-5) were selected to participate in the study, one as the DI experimental school (n=99) and one as the control school (n=90). The two schools were a mile apart and served similar neighborhoods. Approximately 45% were Hispanic, 45% were Black, 7% were Asian, and 3% were Caucasian. The average attendance rate for both schools was 90%. The mobility rate for the two years of the project was 77% for the DI subjects and 70% for the control subjects. At the end of second-grade, scores on the Comprehensive Test of Basic Skills (CTBS) and Metropolitan Achievement Test (MAT) scores were compared for students who learned from the DI model and students who learned from a traditional basal program. Twenty-three DI subjects had participated in the instruction over the full two-year period, and 27 of the control subjects. These students were considered 'stable'. Students who participated in the second year only were considered 'mobile'. The scores for mathematics performance are reported in this brief.

The traditional program (Holt Math Series, Nichols et. al., 1981) was aligned with the standardized test (CTBS) used by the district because the district felt that the test scores had been too low. In an effort to raise standardized test scores, Camden aligned the curriculum with the skills measured by the CTBS-Form U Level D. Curriculum alignment was achieved by listing the skills of the standardized test and comparing them to the skills listed in the curriculum objectives and curriculum materials of the tradi-

Table 1. A Comparison of Percentile Rank Means (Standard Deviations in Parentheses) of Direct Instruction and Control Subjects

Subtest	Direct Instruction		Control		F values		
	Stable n=23	Mobile n=76	Stable n=27	Mobile n=63	Treatment	Mobility	Interaction
CTBS							
Total Math	88(14)	87(19)	87(21)	81(18)	1.30	0.98	0.58
Computation	91(14)	88(18)	82(18)	83(17)	5.22	0.22	0.58
Math Concepts	68(19)	74(20)	85(19)	66(18)	1.21	2.31	7.26**
MAT							
Total Math	76(16)		50(19)		7.30**		
Computation	76(16)		48(18)		8.61**		
Math Concepts	78(18)		48(15)		10.13**		
Problem Solving	61(12)		53(21)		0.60		

*p > .05 **p > .01

tional basal. Where disparities occurred, the curriculum was revised. Teachers were trained to implement the revised curriculum. A timetable for meeting objectives was established. The academic year was divided into quarters and specific curriculum objectives were addressed in each quarter. Supervisors provided assistance and guidance in helping teachers teach content related to the quarterly objectives. All elementary schools in the district used an aligned curriculum except the school chosen to use Direct Instruction. The DI treatment used prepublication versions of Connecting Math Concepts (Engelmann & Carnine, 1992).

On the CTBS and on the MAT, the DI group scored significantly higher on Math Computation than the control group.

At first, the DI teachers were fearful that their students' standardized test scores would fall if they did not follow the aligned program. However, they were assured by the Principal and Assistant Superintendent that they would not be required to meet the district's quarterly objectives. A special DI coach was assigned to help teachers implement the DI programs.

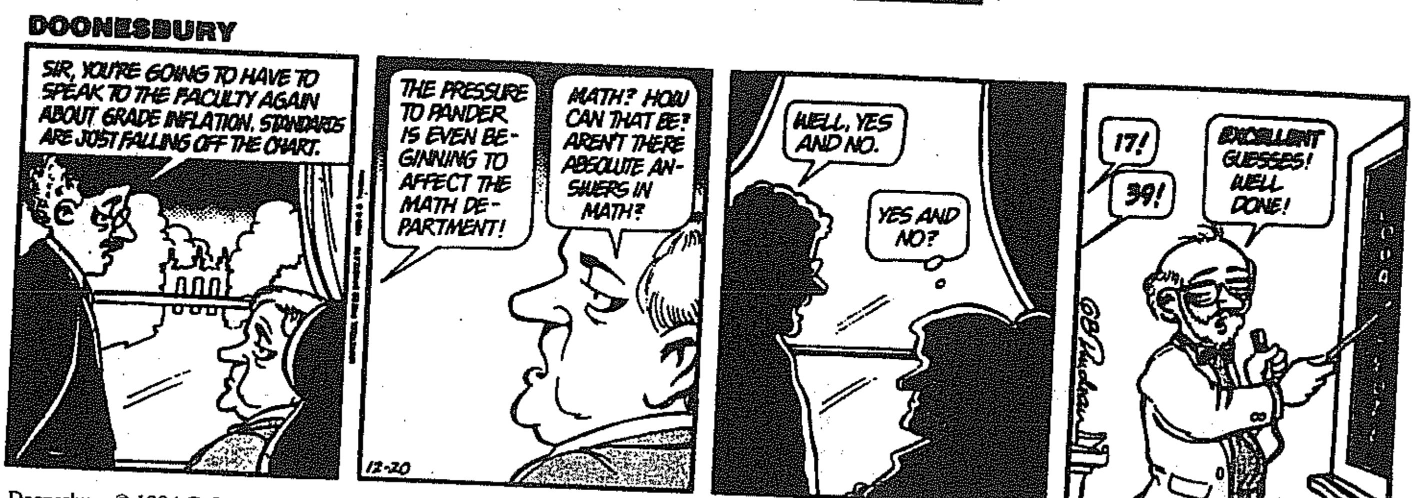
A 2 x 2 analysis of variance was used to analyze each CTBS score. The two factors were (a) program treatment (DI versus control), and (b) mobility (stable versus mobile). Normal curve equivalent scores were converted to percentile ranks. Means, standard deviations, and F values are reported in Table 1. The MAT test was administered to stable students only.

On the CTBS and on the MAT, the DI group scored significantly higher on Math Computation than the control group. However, for the Concepts subtest of the CTBS, there was an interaction. Stable control students scored higher than both DI groups, who in turn scored higher than the mobile control students.

In contrast, DI stable students scored much higher than the stable control students on the MAT concepts test. The alignment of the control curriculum with the CTBS math concepts subtest seemed to contribute significantly to the better performance of the control group on that test, (but benefited only stable students). The higher performance of the DI students on MAT concepts and computation tests suggest that the CTBS results were more dependent on the aligned curriculum, and that the DI program led to more generalized math abilities in computation and math concepts. Furthermore, it seems that mobility had a more negative impact on learning math concepts for the control treatment than it had for the DI treatment.

Instead of having "answers" on a math test, they should just call them "impressions," and if you got a different "impression," so what, can't we all be brothers?

John Handy



Accelerating Cognitive Growth: The Edison School Math Project

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Abstract: *The following study reports the results of a one-year pilot of* Connecting Math Concepts *in the third and fifth grades. Based on the very positive results a full implementation is scheduled for the coming year.*

The Edison Elementary School in Kalamazoo, Michigan is a Kindergarten through sixth-grade public school where 87% of the approximately 600 students are eligible for lunch assistance programs. The school's scores on the state's mandated fourth-grade assessment test for math have been very low for the two years that the current version of the test has been given. In 1992, only 9.7% of the school's fourth-grade students achieved a satisfactory score on the test, compared with the district's average of 30.6% and the state's average of 42.3%.

Over the past few years, many of Edison's teaching staff have complained about the basal math program used in the school (Addison-Wesley, 1985). Specifically, they thought that the program did not help their students to master skills in computation, story problems and fractions. They criticized the spiral design of the program, the absence of systematic review, and the lack of time spent on important content.

This article describes how some of the staff at Edison have been working to improve their students' math skills over the last two years by using the *Connecting Math Concepts* program from SRA. The *Connecting Math Concepts* (CMC) program is a six level (A-F), Direct Instruction basal math program. Each level contains 120 lessons and usually takes one school year to teach. Level A is usually started in the first-grade, Level B in the second-grade and so forth. The *Connecting Math Concepts* program can be taught to large groups of students, assuming that most of the students place into a particular level of the program. For a further description of the CMC program and the objectives of each level, see the CMC Lesson Sampler (available through SRA).

Method

Starting Out

In the 1991-92 school year, two teachers team-taught two fifth-grade classes. As they were to teach math to all of the students, it was decided to divide them into two groups—one higher-performing and one lower-performing. The lower group also contained many students who had social-behavior problems.

When one of the teachers asked the school psychologist for assistance, it was suggested that the teacher use one of the levels of the CMC program. After reviewing the levels available at that time, they decided that CMC Level C would be most appropriate. Pretesting with the Kaufman Test of Educational Achievement—Comprehensive (KTEA-C) Math subtests confirmed that the students were achieving at a third-grade level. Following purchase of the materials and training, the program was started in October 1991, with a group of 25 students.

After eight months of instruction, the students' average gain on both the KTEA-C Computation and Applications Subtests was 1.0 grade equivalent. The students also scored as well or better than other Edison fifth- and sixth-graders on a test containing selected story problems that were common to the CMC Level C program and the third-grade level of the Addison-Wesley program.

Based upon these results and on the teacher's positive experience with CMC, the school decided to use CMC in more classrooms for the 1992-93 school year.

Description Of Classrooms

Two third-grade teachers volunteered to use CMC in their heterogeneously grouped classrooms of approximately 25 students each. Forty students (about 20 in each classroom) placed into Level C. (About five students from each class placed into Level B and were taught by another teacher. The data given later in this article does not include the Level B students' results.) These teachers were each given about six hours of training in using the Level C program and were also observed teaching about four times during the first few months of implementing the program.

The fifth-grade pilot teacher also decided to use CMC with both of her fifth-grade math groups. She used Level C with the lower-performing group and Level D with the higher-performing group. The students were grouped according to their CMC placement test scores and reading ability. Since the teacher was trained in CMC and had been observed several times during the previous year, she received no further training. The school psychologist met with the three teachers about once every two weeks to discuss the students' progress, mastery test results and any implementation problems.

All CMC teachers added a daily point system to the program based upon the students' participation and individual seatwork. This was used all year in the third-grade classes but dropped in the Spring in the fifth-grade classes because the teacher felt it was no longer needed.

The performance of the CMC classrooms was compared with that of several classrooms using a more traditional program (Addison-Wesley, 1985).

Measures

Problem Solving Test

A test was developed for Level C and for Level D. The tests contained story and table problems similar to what the students would encounter near the end of each program. The test items were also selected because they were similar to problems found in the third- and fifth-grade levels of the Addison-Wesley program. These tests were given in May, 1993 to all students in the CMC programs at Edison. The students in the third-grade classes had completed all 120 lessons of Level C when they took the posttest. The students in the fifth-grade classes had completed only 90 lessons (out of 120) when they were given the posttest.

The problem-solving tests were given (also in May, 1993) to three other third-grade classes. One class at Edison had demographics similar to the CMC classrooms. The other two classes were from other schools in which most students have a high socio-economic status (SES). The tests were also given to two other fifth-grade classes (one at Edison and one in a high SES school). All of these comparison classes used the Addison-Wesley math program.

Iowa Test of Basic Skills (ITBS)

The ITBS is given to all students at Edison every year. This year (1993), the students took the test in April. The test was group administered by the students' teachers. The math section of the ITBS contains three subtests: Computation, Concepts, and Problem-Solving. These three subtests are combined to give a Total Math Score.

The results of the ITBS were compared to the previous year's score for each student using CMC for each third-grade class and the high fifth-grade class. The third-grade scores were also compared with three other third-grade classes. (One class from Edison and two classes from another school in the district. All of these classes had similar student demographics and all of them used Addison-Wesley.)

Kaufman Test of Educational Achievement—Comprehensive Form (KTEA-C)

The KTEA-C has two subtests: Computation and Applications. The KTEA-C Computation Subtest is a test of written computation skills ranging from writing numbers and basic facts to higher algebraic skills. This subtest was given in September as a pretest to all students in the four math classes. It was group administered to each class.

The KTEA-C Applications Subtest covers story problems and other math applications (money, time, measurement, etc.). This subtest was also given in September and was given individually because, following standardized procedures, the items had to be read to the students. Since this is extremely time consuming, six students were selected from each classroom to take this subtest. The six included two low-performing students, two average-performing and two high-performing students selected by their teachers.

These subtests were also given in May as a posttest measure. The Computation Subtest was given to all students who had taken the pretest and were still attending the classrooms using CMC. The Applications Subtest was given to the same six students in each classroom who were given this subtest as a pretest. As with the problem-solving test, the third-grade classes had completed 120 lessons, while the fifth-graders completed 90 lessons when they were given this test. The KTEA-C was administered by the school psychologist.

In-Program Mastery Tests

Levels C and D of the CMC program each contain short tests given every 10 lessons. These tests cover the most important content taught in the previous 10 lessons. The passing criteria for the tests are provided in the teacher's manual. If a certain percentage of the students do not meet the criterion for any part of the test, the teacher repeats parts of the previous lessons to review those specific skills. Otherwise, the teacher presents the next lesson, paying close attention to any students who did poorly on the test. All three teachers gave these tests as directed in the manual, although this report only presents class test averages for the two third-grade classes.

Results

Teacher Feedback

All of the teachers who taught the *Connecting Math Concepts* program had very positive experiences with it, even those who initially had serious reservations about Direct Instruction. Teachers often made comments about the high student success rate, increased on-task behavior, sophisticated problem-solving skills, and improved student confidence.

Third Grade

Problem-solving test. The performance of the CMC third-grade classes (means = 64% and 75%) was better than that of a comparable class (mean = 33%). The CMC classes also scored as well or better than students with a much higher SES.

Table 1. Performance of all Third-Grade Classes on the Problem Solving Test.

	Mean % accuracy	% free lunch
CMC classroom #1	64%	87%
CMC classroom #2	75%	87%
Addison-Wesley class, same school	33%	87%
Addison-Wesley, High SES class #1	69%	3%
Addison-Wesley, High SES class #2	46%	8%

ITBS. The scores below reflect the changes in percentile rank from second to third grade for low SES students (approximately 87% on lunch assistance). Students in CMC classrooms seemed to hold their ground, while students learning from a more traditional program were more likely to lose ground.

Students in the quasi-traditional classrooms had extensive experience with "hands on," manipulative-type activities in the first and second grades. In third grade, these students were placed in the Addison-Wesley program. Their overall performance was far below the performance of students in the other three classrooms.

Table 2. Performance of Third-Graders (Low SES) on the ITBS. (Total Math Percentile Rank).

	2nd grade (Pre)	3rd grade (Post)	change in %tile rank
CMC classroom #1	52	49	-3
CMC classroom #2	60	61	+1
Addison-Wesley class, same school	65	50	-15
Quasi-traditional	26	22	-4
Quasi-traditional	34	22	-12

KTEA-C. Pre- to posttest gains on the KTEA-C for the CMC students were all more than 1 year. At the end of third grade, the CMC students were scoring at or above fourth-grade level.

Table 3. Performance of CMC Third-graders on the KTEA-C.

	Pretest-9/92 mean grade equivalent	Posttest-5/93 mean grade equivalent	Gain
CMC Classroom #1			
Math Calculation	3.0	4.5	+1.5 yr
Math Applications	2.9	4.1	+1.2 yr
CMC Classroom #2			
Math Calculation	3.1	5.1	+2.0 yr
Math Applications	3.1	4.5	+1.4 yr

An important question to ask is whether the *Connecting Math Concepts* program is appropriate for high-performing students. Table 4 shows the data for the four third-grade academically talented (AT) students who were in the CMC program:

Table 4. Performance of Academically Talented CMC Third-Graders on the KTEA-C.

	Pretest 9/92 mean grade equivalent	Posttest 5/93 mean grade equivalent	Gain
Math Calculation	3.5	5.7	+2.2 yr
Math Applications	4.1	6.1	+2.0 yr

In-program Mastery Tests. In-program mastery tests, administered every 10 lessons, were used to assess student learning of recently taught content. The mean class averages for the two CMC third-grade classrooms were 88% and 93%, respectively, for the 12 mastery tests.

Fifth Grade

Problem-Solving Test. The mean of the CMC fifth grade from the low SES school far exceeded the mean of a comparable class, and was slightly higher than the mean of a high SES class of fifth-graders.

Table 5. Performance of all Fifth-Grade Classes on the Problem-Solving Test.

	Mean % accuracy	% free lunch
Low SES CMC	82%	87%
Low SES — Addison-Wesley	36%	87%
High SES — Addison-Wesley	79%	8%

ITBS. Comparison data other than percentile ranks for fifth-grade students were not available.

Table 6. Fifth-Grade ITBS Scores—Total Math Percentile Rank.

	4th grade (Pre)	5th grade (Post)	Change in %tile rank
CMC	46	46	none

KTEA-C. The pre- to posttest gains on the KTEA-C showed that the academically talented CMC students generally gained about two years. They were performing at eighth-grade level at the end of fifth grade.

Table 7. Performance of the Academically Talented CMC Students (N = 4) on the KTEA-C.

	Pretest 9/92 mean grade equivalent	Posttest 5/93 mean grade equivalent	Gain
Math Calculation	6.0	8.0	+2.0 yr
Math Applications	6.3	8.5	+2.2 yr

Table 8. Performance of Average and Above Average CMC Students (Level D) on the KTEA-C.

	Pretest 9/92 mean grade equivalent	Posttest 5/93 mean grade equivalent	Gain
Math Calculation	5.1	6.7	+1.6 yr
Math Applications	4.9	5.6	+ .7 yr

Table 9. Performance of Low CMC Students (Level C) on the KTEA-C.

	Pretest 9/92 mean grade equivalent	Posttest 5/93 mean grade equivalent	Gain
Math Calculation	3.9	5.3	+1.4 yr
Math Applications	4.5	4.7	+ .2 yr *

A sample of six students was given the math applications subtest, a test that is individually administered. Four of these six students had a pretest score of 5.2 grade equivalent or higher. Since they were placed in the third-grade level of CMC, the likelihood of these scores going up significantly was slim. These students were placed in the third-grade level because of their very low reading skills, which necessitated placement in the low reading group. The grouping options were limited, and unfortunately, they couldn't be placed in the higher math group. The fifth- and sixth-grade teachers are planning to do more cross-grouping next year in reading and math in order to provide more options for students and to prevent this from reoccurring.

Discussion

The overall pattern of results on the KTEA-C showed that students placed in the *Connecting Math Concepts* program achieved average to above average rates of progress in the areas of math calculation and math applications. This was especially true with the third-grade students, and in general, with students who were placed at their instructional level based upon the placement test.

The problem-solving test comparisons showed that students who learned from CMC could solve sophisticated story and table problems at levels significantly exceeding those of other disadvantaged students and matching or exceeding those of advantaged students.

On the ITBS, the CMC students generally stayed around the 50th percentile while other third-grade disadvantaged students (including students who had been taught with extensive use of manipulatives in grades one and two) experienced significant declines from second to third grade.

These results are particularly impressive given the student population involved. Historically, disadvantaged students in general have performed significantly below grade level on math achievement tests. However, the CMC students either matched or exceeded the performance levels of far more advantaged students who were in more traditional programs.

The results were achieved, we believe, for the following reasons:

1. An extensively field-tested, learner-verified curriculum was used.
2. Teachers received adequate pre-service and ongoing consultation to ensure good implementation.
3. On-going assessments (daily work and in-program mastery tests) of student learning were used to monitor implementation quality.
4. Strong administrative support and encouragement was provided.

The results of this study imply the following:

1. We can accelerate students' cognitive growth in math.
2. Low SES students can match or exceed high SES students in important math computation skills, concepts, and applications when the students are carefully taught, using a validated curriculum.
3. It's important that students are appropriately placed at their instructional level. It's likely that the math applications scores for the fifth-graders would have been higher had they been placed in a more advanced level of CMC.
4. The CMC program appears to be very appropriate for high-performing students.

The Future

As a result of this study, Connecting Math Concepts will be piloted in virtually all grades one through six at Edison School this coming academic year. An important part of the decision to pilot this curriculum was the enthusiasm of the teachers involved in the study. Their positive comments about CMC encouraged other teachers to try the program.

An interesting question for future study is: What levels of achievement will students attain when they are in CMC for more than one year? The positive results for fifth-graders were obtained by completing 3/4 of the fourth-grade level of CMC. As our students move through the upper levels of the program, their skills will grow more sophisticated, and they will place in the fifth-grade level of the program in the fifth grade. We would predict that these students will perform at a higher level than obtained in this study.

Postscript (One Year Later)

For the 1993-94 school year, the CMC program was used at Edison Elementary in one 1st grade class (CMC Level A), three 2nd grade classes (CMC Level B), two 3rd grade classes (1 class used both Level B and Level C, the other class used Level C only), one 4th grade class (Level D), three 5th grade classes and one 5th/6th grade class who cross-grouped their students for math one hour a day (1 class used Level C, 1 class used Level D, and 2 classes used Level E) and two 6th grade classes (Level E).

The staff received one-half day of inservice for the CMC program at the beginning of the school year (10 of the 13 teachers trained had some previous experience teaching Direct Instruction programs). The teachers received very little in-classroom monitoring while teaching the CMC program but most of them consulted during the year with their school psychologist who was very familiar with the programs. The program materials were not available to the school until mid-October resulting in most of the classes not finishing the program by the end of the school year.

Testing Results. The same tests were used to assess the students' math progress, the Kaufman Test of Educational Achievement-Comprehensive-Math Computation Subtest and the problem solving test. Unless noted below, the classes tested were in the Edison school. If CMC students did not complete the entire level, the number of the last lesson completed is shown in parentheses.

Table 10. Results of Second Year.

		KTEA Computation				
Grade Level/ SES	Program	Pretest Grade Equivalent	Posttest Grade Equivalent	Gain	Problem Solving	Lunch Assistance
1/low	CMC Level A	-	2.3	-	-	87%
1/low	Addison Wesley*	1.0	1.5	+.5yr	-	87%
2/low	CMC Level B (90)	-	2.6	-	-	87%
2/low	CMC Level B	2.3	3.5	+1.2yr	-	87%
3/low	CMC Levels B & C	2.5	4.5	+2.0	55%	87%
3/low	CMC Level C	2.9	5.1	+2.2	70%	87%
3/moderate	Saxon	3.4	5.5	+2.1	50%	45%
3/high	Addison Wesley	-	-	-	70%	8%
4/low	CMC Level D (100)	-	6.0	-	79%	87%
4/low	Addison Wesley	4.1	5.1	+1.0	30%	87%
4/high	Addison Wesley	-	-	-	55%	8%
5/low	CMC Level E (100)	-	8.0	-	87%	87%
5/high	Addison Wesley	-	-	-	55%	8%

*Supplemented with additional teacher-made materials.

The data collected this year was overwhelmingly positive with regards to the *Connecting Math Concepts* program. The prediction made in our original article that the difference in scores on the problem-solving test between CMC students and students in more traditional programs would increase as the CMC students advanced through higher levels was largely borne out. This is due, we believe, to the incremental approach of CMC. Concepts, operations, and applications are carefully and cumulatively developed over time as opposed to the spiral approach taken in traditional programs where topics are repeated year after year, resulting in less new learning, particularly at the upper elementary level.

As reported in our original article, 1993-94 was to be a pilot year for the Edison Elementary (where 85% of the students used the CMC program) as well as for the entire Kalamazoo Public School elementary levels, where other math programs were also tried out. The results reported here, along with very positive teacher and parent reports, led to the Edison teaching and administrative staff supporting the continued implementation of the CMC program. Since the remainder of the elementary schools in the Kalamazoo system will be using another program (based upon the district math committee's recommendation), the staff at Edison made a special request to the math committee's chair to continue using CMC. This request was granted. For next year, almost all of the the 1st through 6th grade classrooms at Edison will be using the CMC program.

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Evaluating a Mathematics Program for Adoption: Connecting Math Concepts

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Upper Darby School District, Pennsylvania

The Upper Darby School District in Upper Darby, Pennsylvania is a suburb of Philadelphia, PA. The school system consists of eight elementary schools, two middle schools and one high school. The school population is approximately 10,000 students, representing a diverse socio-economic and ethnological background. There are three full-time English as a Second Language (ESL) classes at the elementary level, two at the middle school level and one at the high school level serving students representing over 35 nationalities. Also, the stability (students starting at the earliest grade and completing that sequence) of the student population fluctuates between 30% and 40% throughout the elementary, middle, and high school levels.

In order to control budget and maintain educational programs, the district has adopted a curriculum organizational structure consisting of five year cycles for the basic programs and six year cycles for the related arts programs. There are Curriculum Study Committees for the elementary, middle school and high school levels, consisting of two teachers per grade level, an administrator and a representative from special education.

One of the functions of these committees is to explore educational programs which will enhance the opportunities for learning. Committee members attend workshops and seminars where they learn of and receive training in current educational programs. One such workshop, which was attended by district personnel, was presented by Bernadette Kelly, Ph.D., Research Associate from the University of Oregon, who discussed the problems intrinsic in traditional basal programs that have been eliminated in the Direct Instruction programs.

Through the guidance of Science Research Associates and relying upon the expertise of Dr. Kelly, the Upper Darby School District ran an experimental program using *Connecting Math Concepts* (CMC) in first and fourth grades during the 1992-93 school year. To use a Direct Instruction program in fourth grade was a most ambitious undertaking because it represented a complete reversal of the students' first three years of mathematical experience.

Organization of the Study and Analysis of Results

One teacher from the first and one from the fourth grades at each of the elementary schools was asked to volunteer for the program. Each teacher attended a one day workshop to receive training in the CMC program. The background of these volunteers included a first year teacher as well as teachers who had over twenty years of classroom experience.

In order to determine growth, a pretest was given to each class of students in the pilot program and to a comparable class using the basal series. The pretest was the one accompanying the CMC program which was designed to determine the readiness of the student for that level. There were no significant differences between the means of comparable classes on the pretests.

The posttests were designed by the teachers involved in the pilot study. They were instructed to select concepts which were common to both groups and present them in a neutral format. Instructions were provided using the terminology that was used in each program. These tests were administered without time constraints so that all students had the opportunity to demonstrate their understanding of the concepts without the additional pressure of specific time limitations. These tests were administered during the first week of February.

Measures of reliability were calculated for the pre- and posttests using the Kuder-Richardson Formula No. 20. These measures at the first grade level were very low on the CMC pretest (0.38) but were much improved on the teacher-made posttest (0.86). The low reliability on the pretest can be explained by the limited number of problems and the narrow range of the results. The posttest had more items and the results represented a wider range therefore a higher reliability coefficient was produced. District and school analyses can be seen in Tables 1 and 2 for pre- and posttest results.

Table 1. Descriptive Statistics for Pretest: First Grade.

	Total N	Total Mean	Total StDev	Pilot N	Pilot Mean	Pilot StDev	Control N	Control Mean	Control StDev
District	323	9.70	0.72	164	9.66	0.76	159	9.74	0.68
Elem #1	40	9.65	0.83	20	9.70	0.66	20	9.60	1.00
Elem #2	46	9.59	0.91	25	9.48	0.92	21	9.71	0.90
Elem #3	39	9.64	0.63	22	9.77	0.53	17	9.47	0.72
Elem #4	36	9.75	0.77	14	9.50	1.09	22	9.91	0.43
Elem #5	42	9.83	0.54	22	9.86	0.35	20	9.80	0.70
Elem #6	41	9.56	0.81	23	9.44	0.99	18	9.72	0.46
Elem #7	43	9.93	0.26	20	9.90	0.31	23	9.96	0.21
Elem #8	36	9.61	0.77	18	9.61	0.85	18	9.61	0.70

Table 2. Descriptive Statistics for Posttest: First Grade.

	Total N	Total Mean	Total StDev	Pilot N	Pilot Mean	Pilot StDev	Control N	Control Mean	Control StDev
District	323	27.99	4.71	164	28.20	3.99	159	27.78	5.36
Elem #1	40	27.85	4.20	20	27.95	3.10	20	27.75	5.16
Elem #2	46	28.24	4.37	25	28.64	4.92	21	27.76	3.66
Elem #3	39	27.41	5.04	22	27.23	4.62	17	27.65	5.67
Elem #4	36	23.75	5.79	14	26.21	4.53	22	22.18	6.05
Elem #5	42	30.60	2.47	22	30.41	1.99	20	30.80	2.95
Elem #6	41	26.20	5.44	23	26.00	3.69	18	26.44	7.21
Elem #7	43	29.40	3.33	23	29.75	3.11	23	29.09	3.54
Elem #8	36	30.00	2.72	18	28.94	3.33	18	31.06	1.31

Table 3. T-scores for Pre- and Posttests: First Grade.

	Pretest	Posttest
District	-0.97	0.80
Elem #	10.37	0.15
Elem #2	-0.85	0.68
Elem #3	1.42	-0.24
Elem #4	-1.29	*2.21
Elem #5	0.36	-0.49
Elem #6	-1.20	-0.23
Elem #7	0.61	0.02
Elem #8	0.00	*-2.43

*sig > .05 level

A visual inspection of this pretest data indicated that the initial abilities of these groups were not significantly different. However, in the posttest two schools did produce differences which were significant beyond the .05 level—one in favor of the pilot group (ES#4) and the other in favor of the control group (ES#8). The t-values for pre- and posttest results are in Table 3. A positive t-value indicated that the pilot mean was greater than the control mean.

There were many variables which were not controlled, including but not limited to each pilot teacher's acceptance of Direct Instruction strategies, particularly by those teachers who had been developing mathematics through discovery with manipulatives. Even though these teachers volunteered to pilot the program, their level of acceptance did vary. Some were not comfortable with the scripted format and others had difficulty accepting the application of manipulatives as reinforcement of learning rather than using manipulatives for student discovery.

Most teaching strategies advocate the transition from concrete to abstract which does seem reasonable and plausible. To accept alternative strategies and incorporate these in one's teaching philosophy requires unique teachers who are searching for means by which their students will become better learners. How these adjustments are handled in adopting a Direct Instruction program will be crucial to the success of the program.

The fourth grade results were much more dramatic (see Tables 4 and 5). This can be attributed but not limited to the greater reliability of the testing instruments. The pretest developed by CMC had a reliability coefficient of 0.81 and the posttest developed by the teachers produced a reliability level of 0.94. The students in the pilot group were significantly lower in ability according to the pretest. No

RESEARCH

elementary school had a pilot group that had significantly greater initial ability than the control group, whereas three of the pilot groups were determined to be of significantly lesser ability than the control group.

Instruction using the CMC mathematics program reversed these results significantly in all but two schools. By February, students in six of the eight pilot groups were outperforming those students in the control groups. In one school (ES#8), the pilot group had begun the year significantly below the control group and was making gains in reversing their differences as reflected in the change in t-test scores from -2.10 to +1.52 (see Table 6).

The other school (ES#4) did appear to narrow the gap between the pilot and control groups. What was unique about this effort was the fact that the pilot group was the lowest ability level of three groups in an open classroom setting and the control group was the highest. Usually, one would expect that if two groups begin at different levels of ability that the distance would widen as they further progress in the educational process.

In a study analyzing the DI data from Project Follow Through (Abt, 1977), the growth within levels of ability as measured by the Metropolitan Achievement Test was not significantly different for Direct Instruction students at each ability level. Lower ability students exhibited the same amount of growth as those of higher ability (Gersten, Becker, Heiry, and White, 1984). The results from ES#4 seem to support this data because the gap narrowed rather than widened between these groups.

The teacher of the pilot program in that school reported that all of the students that had been receiving Chapter 1 services in mathematics tested out of the Chapter 1 program at the end of the year.

Discussion

This study seems to indicate that the Direct Instruction teaching process has been most effective at the upper level. There seems to be no clear-cut reason as to why these results were not duplicated at the primary level. One reason might be found in the level of complexity of material since the first grade concepts represent a narrower scope of material as compared to the breadth of concepts at the fourth grade level.

Table 4. Descriptive Statistics for Pretest: Fourth Grade.

		Total			Pilot			Control	
	N	Mean	StDev	N	Mean	StDev	N	Mean	StDev
District	360	27.70	4.28	181	27.00	4.13	179	28.50	4.31
Elem #1	49	28.70	4.13	25	27.80	4.32	24	29.70	3.44
Elem #2	46	23.93	4.89	23	24.70	5.51	23	28.60	3.27
Elem #3	50	27.80	4.28	25	28.00	3.43	25	27.50	5.05
Elem #4	36	27.60	4.71	17	23.90	4.12	19	30.90	1.78
Elem #5	41	28.90	3.86	20	28.60	2.06	21	29.20	5.06
Elem #6	41	25.00	5.18	21	25.40	3.85	20	24.60	6.36
Elem #7	51	28.40	2.93	27	28.70	3.09	24	28.00	2.77
Elem #8	46	28.40	2.93	23	27.50	3.83	23	29.30	2.12

Table 5. Descriptive Statistics for Posttest: Fourth Grade.

		Total			Pilot			Control	
	N	Mean	StDev	N	Mean	StDev	N	Mean	StDev
District	360	46.30	7.47	181	49.60	4.60	179	43.00	8.34
Elem #1	49	49.20	5.60	25	50.80	4.01	24	47.50	6.53
Elem #2	46	41.20	8.45	23	47.50	4.80	23	34.90	6.27
Elem #3	50	43.20	10.50	25	51.30	3.99	25	35.10	8.55
Elem #4	36	49.70	5.45	17	45.60	4.87	19	53.40	2.48
Elem #5	41	47.20	5.44	20	49.50	5.45	21	44.90	4.46
Elem #6	41	44.90	8.06	21	49.70	3.60	20	39.80	8.45
Elem #7	51	48.20	5.28	27	51.90	2.74	24	44.00	4.18
Elem #8	46	47.80	4.15	23	48.70	4.72	23		

Table 6. T-scores for Pre- and Posttests: Fourth Grade.

	Pretest	Posttest
District	-3.33*	9.19*
Elem #1	-1.62	2.13*
Elem #2	-2.83*	7.52*
Elem #3	0.39	8.39*
Elem #4	-6.35*	-5.80*
Elem #5	-0.44	2.90*
Elem #6	0.52	4.70*
Elem #7	0.84	7.75*
Elem #8	-2.10*	1.52

During the 1992-93 school year, the Elementary Curriculum Study Committee developed end-of-year tests for grades one through five. The purpose of these tests was to allow us to evaluate curriculum and determine areas of deficiency. The results were analyzed and student mastery was defined to be at the 70% level. These results indicated that the higher the grade level, the lower the rate of mastery (see Table 7). These results do include the pilot groups. Their inclusion might be one reason for the lack of continued decline in results between grades three and four as exhibited between other grade levels. These results did indicate that a change in curriculum was needed.

Based on the positive results of the pilot implementation, a decision was made to adopt *Connecting Math Concepts* district-wide in grades 1 through 5, beginning in the 1993-1994 school year. The following ideas represent areas which our district has had to address in adopting a Direct Instruction program.

1. *Ownership of instruction within the classroom.* In public education, everyone is an authority because they have been "through the system." This attitude is very much in evidence whenever the School Board attempts to develop and implement budgets, programs, etc. The "authorities" come out of the woodwork and know "what's best." This is no different with teachers. No matter what methodology might be most effective, the "best" method is what I, the teacher, have been doing for the past twenty years. This is a most difficult attitude to overcome and a most serious problem for mathematics -- teachers tend to teach mathematics in the same manner that they were taught. As all administrators realize -- once the classroom door is closed, the program is at the mercy and interpretation of the teacher. This ownership in presentation has been most difficult for some teachers to relinquish.

2. *Material not challenging.* Teachers perceive that the work is too easy. This reaction was most interesting because a comparison of the existing curriculum and the CMC program revealed very few differences except within and/or between grade levels. There was a difference in the sequencing of concepts during the school year but not in the nature of the concepts. In some cases, material presented in the CMC program was presented at an earlier or later grade level as compared to the existing curriculum. If the material of the CMC program was consistent with the existing program but was being perceived as being too easy, then the possibility exists that student success in understanding might be contributed to the logical development of concepts through Direct Instruction.

There also exists a difference in what teachers might perceive as levels of difficulty. Given the choice of (a) 3(2 + 4) or (b) 3 2/3(12.75 - 9 1/2) as examples for testing understanding of the distributive property, many teachers might select example *b* for higher ability groups because it represents the "harder" problem. However, what these teachers have not realized is that these questions are not measuring the same concept. The second choice does use the distributive property but analysis of that understanding has been distorted by the arithmetical gymnastics of the fractions and decimals. Example *b* does not take mathematics to a higher level of understanding.

After many discussions regarding the philoso-

Table 7. End of Year Results on District Tests.

Grade	N	Mean	St>70%	%>70%	St>90%	%>90	Probs
First	782	62.7	750	95.9%	504	64.5%	69
Second	768	61.0	597	77.7%	206	26.8%	76
Third	729	42.0	413	56.7%	98	13.4%	59
Fourth	654	40.3	370	56.6%	91	13.9%	56
Fifth	643	25.4	132	20.5%	23	03.6%	50

phy of the CMC program, this researcher has come to the conclusion that some teachers believe that there must be a wide range in the grades that students earn in mathematics. This implies that a difference in teacher expectations exists based upon student ability. This philosophy appears to be inconsistent with other subjects, where the goals which most teachers establish for their students are for those students to perform at an A or B level. Somehow these goals change when it comes to mathematics. Teachers have come to expect that their students will exhibit a wider range of abilities and mastery.

The solution to part of this problem would be to have greater homogeneity in the mathematics classroom. CMC has labeled their programs A, B, C, D, E, and F which might imply first grade, second grade, third grade, and so forth. This is not necessarily true because it is possible for students to complete a level before the end of the year and begin the next level. This has proved difficult to incorporate since over 50% of our classrooms at the elementary level are heterogeneously organized which prevents this type of acceleration.

3. ***Teacher perception of what students are capable of learning.*** The philosophy implied by Direct Instruction of allowing all students exposure to the same material agrees with the Standards developed by the National Council of Teachers of Mathematics (NCTM) which indicate that all students must have the same opportunity to learn. NCTM does recognize individual differences of ability but stresses the opportunity for equal access to learning.

Many teachers, particularly those at the upper levels, have commented that in the past they had not considered teaching a concept because they did not believe their students were capable of understanding. Direct Instruction becomes a valuable tool for preventing preconceptions as the basis for decision making in the mathematics classroom.

4. ***Gradual adoption.*** Due to budget constraints and curriculum organization, our district had to adopt this program in one year. Due to the significant differences in presentation of concepts as compared to traditional programs, consideration should be given to introducing this program over a period of time through volunteer teachers. Once others begin to see the success that can be obtained, then they will enter the process with a more favorable attitude and a favorable attitude is one of the most important ingredients for implementation.

Table 8. Performance of Students in CMC Mathematics Program.

Grade	Number	Number Above 70%	Percent Above 70%
1	887	800	90%
2	800	762	95%
3	817	750	92%
4	724	648	90%
5	697	588	84%
Total	3925	3548	90%

Follow Up

As a follow-up to the implementation of the CMC program, a survey was developed in February, 1994 to determine how each teacher was progressing. Table 8 lists the mastery by grade level for the district at a 70% performance level. If the indicated rates of success are accurate, then we can conclude that over 90% of our students are functioning above the mastery level. This is vastly different from our end-of-year results last year. This data reflects the performance by our students on the tests that are part of the CMC program.

In addition, the teachers were asked to evaluate how well those students that had been in the CMC pilot study last year were performing this year. They were to compare the performance of the student with the performance of the class. Most teachers did analyze their students in this manner. However, some stated that they had heterogeneous classes and had evaluated their students according to the expectations that they, the teachers, had regarding those students. These differences in interpretation did make it difficult to analyze current success (see Table 9).

Table 9. Teacher Evaluation of Level of Performance of CMC Students the following year.

Grade	Total N	Number Working Above Group	Number Working Same Level	Number Working Below Group
2	142	38 (26.8%)	81 (57.0%)	23 (16.2%)
5	166	47 (28.3%)	91 (54.8%)	28 (16.9%)

These results seem to indicate that the CMC students from the pilot program were continuing to find success. No information was gathered regarding placement in special education programs.

Some interesting evaluations of initial success through the end-of-year tests that are being planned include: (1) a comparison between the heterogeneous classes and the homogeneous classes, (2) students in the pilot program with two years of experience as compared to those with one year of experience, and (3) the effect of teacher attitude on student learning. These results will form the basis of our long-range planning in mathematics.

Conclusion

Our district has experienced a wide range of reactions from teachers. The scripting of lessons has been most difficult for some to accept but it is a necessary part of the program. Scripting provides consistency in presentation and the means by which a variety of concepts may be developed in a given class period. For this reason, a strong recommendation would be to implement any Direct Instruction program with teachers who demonstrate both interest and willingness to provide an educational experience that has the potential for significantly improving the mathematical abilities of all students.

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The Effectiveness of Videodisc and Traditional Algebra Instruction with College-Aged Remedial Students

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The major purpose of this study was to compare the effectiveness of System Impact's videodisc program, "Mastering Equations, Roots, and Exponents," with intensive conventional instruction using a standard introductory college algebra text.

Method

Subjects

Subjects in this study were all participants in the "Project Success" summer transition program for learning disabled adults.

Materials

The control group used the *Intermediate Algebra* text by Pulsinelli and Hooper (1991). Because of the time constraints of the summer program, only the first three chapters of the book were covered. Major topics included operations with signed numbers, solving equations and inequalities, and roots and exponents. The treatment group (SI) used the Systems Impact *Mastering Equations, Roots, and Exponents* videodisc program (1989). Major topics included in the videodisc program included roots, exponents, and solving equations.

Procedures

During the first week of the eight-week summer program, the videodisc program's placement test was administered to 43 students. Twenty-six students scored in the "ideal" range for the study in that they had a good grasp of basic operations with decimals and fractions, but lacked basic algebraic concepts such as solving equations and the rules of exponents. From the subject pool of 26 students, 13 students were randomly assigned to two groups. One group learned from the Systems Impact videodisc program (SI). The other group learned from the algebra text (Text).

During the first two days of classes, students in each group were given the Pulsinelli and Hooper Chapter 1–3 summary examination drawn from the textbook program's multiple choice test bank. The videodisc program's pretest was also administered. Analysis of the videodisc program's pretest results revealed no significant difference between the two groups. In contrast, on the Pulsinelli and Hooper examination, the Text group scored significantly higher than the SI group.

Both the SI and Text groups received one hour of instruction daily over 28 class days. The Text group instructors were experienced teaching assistants who had worked with the Pulsinelli and Hooper materials over the past year and who had successfully completed the rigorous, 70-hour training program for instructors in Project Success. In addition, the instructors were given technical assistance in direct instruction techniques specific to mathematics and in the use of concrete and semi-concrete manipulatives.

Two instructors were assigned to the Text group to give better attention to individual student's needs both during class periods, and, when necessary, outside class during study periods.

One instructor taught the SI group following the instructions and suggestions in the teacher's guide for the videodisc materials. In general, each videodisc lesson lasted 40–45 minutes. Four days each week, the remainder of the 60-minute class period was used to relate the videodisc topics to the Pulsinelli and Hooper topics. For example, the videodisc series does not cover solving and graphing inequalities; however, this topic was covered during several of these 15-minute sessions after the videodisc lesson. In all, 23 of the videodisc program's 30 lessons were presented to the students during the six-week session.

During the last two days of the sixth week of the program, both groups were again given the Pulsinelli and Hooper Chapters 1–3 *Intermediate Algebra* test and the posttest for the videodisc program.

Results

Table 1 below provides pre-instruction means for the SI (n = 13) and Text (n = 13) groups on the following variables: WRAT Reading and Spelling subtest scores, Stanford Diagnostic Reading Test (SoRT) comprehension subtest score, Slosson IQ score, and age. No significant differences were found in any of these areas.

Table 1. Pretest Scores for Videodisc and Control Groups.

	WRAT Read	WRAT Spell	SDRT Comp	Slosson IQ	Age
SI group	55.38	24.03	32.05	95.31	19.3
Text group	52.14	21.86	31.96	96.33	19.0

Figure 1 and Table 2 display the groups' mean scores on the pre- and posttest criterion-referenced measures used in this study. On the videodisc pre- and posttest, the SI group made a gain of 15, while the Text group made a gain of 3.8. On the *Intermediate Algebra* tests, the SI group made a gain of 10.2, while the Text group made a gain of 3.2. The differences in test scores favored the SI treatment and were significant in terms of widely accepted criteria for practical and statistical significance.

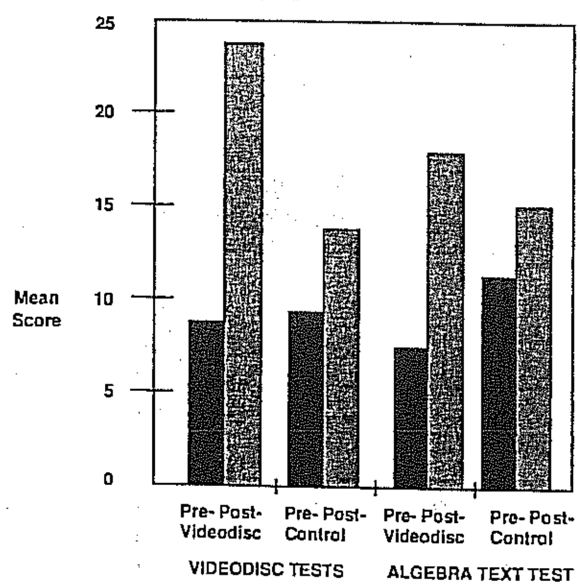

Figure 1. Pre- and Posttest Means

Table 2. Pretest – posttest means, Standard Deviations and t-values for the videodisc and control groups.

Measure	Pretest	t-value	Posttest	t-value
Videodisc test				
SI group	8.77 (3.81)		23.38 (5.28)	
Text group	9.23 (3.44)	.032 (p = .75)	13.00 (5.74)	4.80 (p = .001)
Text test				
SI group	7.54 (3.48)		17.77 (2.28)	
Text group	10.92 (3.68)	2.41 (p = .024)	14.15 (4.20)	2.73 (p = .014)

Discussion

Before beginning this study, we expected to find a significant difference between the two groups on the videodisc posttest results in favor of the SI group. The results of the study verified our expectations. In contrast, we did not expect to find significantly different results in favor of the SI group on the intermediate algebra test from the Pulsinelli and Hooper book. Given that the Text group scored significantly higher on the intermediate algebra pretest, it seemed unreasonable to predict a complete reversal of significance on the posttest as occurred.

There are several major reasons, in our opinion, why the Systems Impact videodisc program proved to be effective with these students. First, the videodisc instruction was presented in strands which allowed for the distributed practice necessary to ensure mastery of the topics within the available time. Second, the task and concept analyses for each topic were first-rate. These analyses generated a curriculum sequence that provided students with the consistent success that they seemed to need, as well as concept generalization. Third, the videodisc instruction student workbooks for the videodisc program provided an ample amount of appropriate practice exercises for the students to complete independently. Fourth, the videodisc program's built-in monitoring and branching system helped the instructor pinpoint students' strengths and weaknesses and to tailor additional instruction to meet individual needs in a timely manner. This additional instruction was achieved by using the alternate examples and teaching activities stored on the discs. The consistently high levels of student success and of conceptual understanding were ideal for these students entering the program with a negative attitude toward math and with severe skill deficits.

Conclusion

Given that the Text group was using a highly regarded curriculum, implemented by a well-trained team of instructors, the superior results attained by the videodisc group attest to the value of the videodisc program for adults in need of an effective remedial program.

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Raising Mathematics Problem-Solving Performance: Do the NCTM Teaching Standards Help? (Final Report)

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Abstract: *We conducted a study to compare the effects on mathematics problem solving of a User-Friendly Direct Instruction mathematics programs with the effects of programs that implement the NCTM teaching standards. A preliminary report was printed in the summer, 1992 issue of the ADI News. That report evaluated whether User-Friendly Direct Instruction was for low performing students only. Those findings are briefly reviewed here. This is the final report, which focuses on evaluating the effects of the two treatments on problem-solving performance after a year and a half of instruction.*

Student performance standards require a different development process than teaching standards require. The development of student performance standards, which describe desirable student learning outcomes expected from schooling, requires a political process reflecting a consensus of what our society views as significant and socially valued learning. On the other hand, the development of teaching standards, which describe teaching practices, requires a scientific process that uses empirical evidence to identify those practices that are superior in achieving the student performance standards.

The Standards Commission of the National Council of Teachers of Mathematics (NCTM) has blurred the distinction between these two types of standards, confusing the means with the ends of teaching. Recommendations for specific teaching practices are interwoven with descriptions of the tasks for student performance standards in the *Curriculum and Evaluation Standards* (NCTM, 1989) and in the *Professional Standards for Teaching Mathematics* (NCTM, 1991). As far as student performance standards are concerned, the content of both documents seems to emphasize problem solving, particularly of non-routine problem types, as an important goal for mathematics instruction. However, the expected levels of performance that would make the identified tasks into "student performance standards" are missing. The NCTM standards seem more influential as teaching standards rather than as student performance standards.

Whether the NCTM Standards Commission is the appropriate political entity to establish the student performance standards for this nation is not an issue that will be discussed here, though it may be a legitimate question. For the purpose of the following evaluation, we accept problem-solving as identified by the NCTM as the important focus for student performance standards. The following study was designed to empirically evaluate the effectiveness of the NCTM's recommended teaching practices in achieving these learning goals.

The NCTM teaching practices (1989, 1991) are based on constructivist theory (Cobb, Yackel, & Wood, 1992; Resnick, 1988), rather than on experimental research. Constructivist theory states that meaningful understanding cannot be transferred from teacher to child, rather children must construct their own meanings through interaction with objects and other children in their natural environment. The NCTM teaching practices are most accurately defined by what they are not; they are not "traditional" teacher-directed instruction. The NCTM teacher does not rely on a textbook and worksheets. The NCTM teacher does not stand in front of a class, with desks lined up in rows, explaining mathematics while children silently listen. The NCTM teacher does not test students on algorithms and facts that students memorize without fully understanding them. The NCTM teacher does not drill students on isolated math skills, but rather presents authentic problem situations from which students gain a wholistic understanding of mathematics. The NCTM teacher places students in heterogeneous learning groups, designs learning activities that makes extensive use of manipulatives, and varies

RESEARCH

the instructional format as much as possible to maintain a lively, interesting class environment. Learning activities emphasize nonroutine problems, which are problems that cannot be solved with an algorithm.

Another alternative to traditional direct instruction is user-friendly Direct Instruction, which derives from the findings of experimental research. Yates and Yates (1990) synthesized experimental research to identify "user-friendly," instruction that is most effective with the widest range of learners. In another review of the research literature, Carnine, Jones, and Dixon (in press) define the state-of-the-art in research-based "effective mathematics instruction for diverse learners." The report of their findings will appear in a special edition of School Psychology Review featuring research-based practices for teaching diverse learners (Carnine & Shinn, in press). A simplified version of that article written specifically for practitioners in the process of selecting mathematics curricula appears in this issue ("Research-Based Guidelines for Selecting a Mathematics Curriculum" by Bob Dixon). Carnine, Jones, and Dixon's findings are consistent with and overlap much of what Yates and Yates (1990) have also

Traditional direct instruction and the NCTM teaching practices may be viewed as two extremes on a continuum with user-friendly Direct Instruction lying somewhere between, balancing aspects of both more extreme approaches.

identified as "user-friendly" instruction. We call the instruction identified by Yates and Yates and by Carnine, Jones, and Dixon "user-friendly Direct Instruction." User-friendly Direct Instruction is entirely consistent with the "effective teaching practices" popular in the previous decade (Brophy & Good, 1986; Rosenshine, 1986). However, user-friendly Direct Instruction is more comprehensive and refers not only to the teacher's classroom behavior, but also to the design of the instructional materials (Carnine, Jones, & Dixon, in press).

Table 1 contrasts user-friendly Direct Instruction with traditional direct instruction and with the NCTM teaching practices. Traditional direct instruction and the NCTM teaching practices may be viewed as two extremes on a continuum with user-friendly Direct Instruction lying somewhere between, balancing aspects of both more extreme approaches.

User-friendly Direct Instruction emphasizes teaching "big ideas," strategies and principles that have wide applicability. Problems that can be solved with an algorithm (e.g., addition, subtraction, multiplication, or division) are a big idea that is based on a thorough understanding of number families (see Dixon, p. 47, and Kelly, p. 11, in this issue). The teacher presents these big ideas in a highly interactive format that allows for explicit but brief initial presentation of the underlying skills and concepts, then scaffolds the students' work as they solve problems using these skills and concepts. The teacher gradually removes the scaffolding as students become more independent working the problems. The teacher or the curriculum carefully prepare students by teaching all the prerequisite knowledge. Practice is sufficient to allow mastery, distributed over time, varied in different contexts, and cumulative. The curriculum is sequenced so that new learning builds on and integrates earlier learning into more complex understandings.

Very little experimental research has been conducted to test whether the teaching practices promoted by the NCTM will help the nation reach the achievement goals established by the same NCTM (Bishop, 1990). Bishop, a prominent writer on research method, pointed out that "recommendations and exhortations [in the NCTM standards] appear to be supported only by opinion—authoritative opinion, it is granted—but opinion nevertheless" (p. 357). The NCTM document openly states that "one reviewer of the Working Draft of the Standards suggested the establishment of some pilot school mathematics programs based on these *Standards*..." (p. 253, 1989), thereby acknowledging that the *Standards* were untested.

The *Standards* are upfront about being "a new research agenda" rather than a description of the state-of-the-art in mathematics instructional research, as many assume them to be. In spite of the lack of experimental research, the NCTM standards are being widely implemented as *teaching* standards, as though they were validated, and these constructivist practices are driving the current design of the nation's basal mathematics programs.

Few may actually realize the educational risks involved in testing the NCTM teaching standards through a nation-wide experiment. The educational risks are particularly heightened by the fact that constructivist theory contradicts the "user-friendly explicit instruction" that experimental research has found most effective in general education and in special education. Sweller (1990) voiced a word of caution: "I do not believe we should be introducing curriculum changes of the sort advocated purely on the basis of theoretical analyses. Proper theorizing is followed up by extensive empirical work to determine the strengths and weaknesses of the theory. Depending on the results, we then may be in a position to advocate curriculum change" (p. 414).

Only two experimental studies have evaluated the effectiveness of constructivist instruction on mathematics learning (Cobb et al., 1991; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). Both these studies are flawed. In a full-year, second grade study, Cobb et al. (1991) found that constructivism resulted in better mathematics achievement than "traditional direct instruction" taught using the Addison-Wesley (1987) second grade textbook. However, there were two serious confounds in this study. First, the teachers administering the constructivist treatment self-selected themselves by participating in the summer institute where they received their initial training. Those teachers who did not participate became the control teachers. It is not clear whether the control teachers even knew their students would be tested. The second confound was that the "extensive support throughout the school year," (p. 14) received by the experimental teachers alone may have had a positive effect on learning, regardless of the nature of the support.

Cobb et al. (1991) cite a second study by Carpenter et al. (1989) as one that resulted in conclusions similar to theirs. The Carpenter et al. study was in fact free from the confounds noted in the Cobb et al. study. Teachers were randomly assigned to the constructivist and control treatments and inservice "support" training was provided for both groups. However, a different confound was present in the Carpenter et al. study. The constructivist inservice workshops focused "on story problems that were relevant to the children, [while] the problem-solving emphasis of the control group's workshop was on mathematics problems that were intriguing and of a more esoteric nature. Such problems are often designated as 'nonroutine'" (p. 507). The use of routine problems in the more effective treatment contradicts the NCTM's recommendation to include a high percentage of nonroutine problems (problems that cannot be solved by an algorithm). The problem-solving standard in the NCTM curriculum and evaluation standards strongly emphasizes nonroutine problems. In view of this fact, the findings of the Carpenter et al. study cannot be viewed

Table 1. A Comparison of Traditional Direct Instruction, User-Friendly Direct Instruction, and the NCTM Teaching Practices.

Traditional direct instruction	User-friendly Direct Instruction	NCTM Teaching Practices (Constructivism)
Emphasize knowing.	Emphasize applying knowledge to solve problems.	Emphasize gaining knowledge through problem solving.
View knowledge as Absolute. Factual. Consisting of unrelated facts.	View knowledge as Expanding. Heirarchical. Consisting of "Big Ideas" that apply to authentic contexts.	View knowledge as Relative. Wholistic. Gained from authentic contexts.
Group by age.	Group by skill level.	Group to mix ages and skills.
Explicit, Teacher-directed.	Explicit, Teacher-directed.	Discovery, Student-directed.
Lecture. Non-interactive. Teacher is active. Students are passive.	Experiential—(Minds-on). Interactive with the teacher. Teacher is active. Students are active.	Experiential—(Hands-on). Interactive among students. Students are active. Teacher is nonintrusive.

as entirely supportive of the NCTM standards.

Furthermore, both studies compared constructivist practices with practices that are not representative of the state-of-the-art in empirically defined classroom practice. Traditional instruction does not equate with research-based, user-friendly Direct Instruction. No research has compared constructivist practices, as recommended by the NCTM, with "user-friendly" instruction (Carnine, Jones, & Dixon, in press; Yates & Yates, 1990).

The following study compared the effects of user-friendly Direct Instruction in mathematics (DI) with instruction that applied the NCTM standards (NCTM) on mathematics problem-solving performance. The Systems Impact series of videodisc mathematics programs were used in the user-friendly DI treatment. These programs have been validated by the National Diffusion Network, which requires that claims of effectiveness be supported by empirical evidence. Inclusion in the National Diffusion Network means that a program has met high standards of demonstrated effectiveness.

During the first year, the NCTM teaching practices were implemented district-wide and in the experimental NCTM classrooms. The DI treatment was implemented in both fifth and sixth grade. The results for the sixth grade group, which were reported earlier in the summer, 1992 issue of the *ADI News*, are briefly reviewed below. The fifth grade treatment group began using the DI treatment in January of the first year and continued through sixth grade, where more complete problem solving assessments occurred. The differential effects on the problem solving of the groups that started the differential treatments in fifth grade comprises the main body of this report.

Is User-Friendly Direct Instruction Only for Low Performing Students?

During the first year of research, the primary research question was whether user-friendly Direct Instruction was appropriate only for low-performing students. Because the strongest research support for user-friendly Direct Instruction seems to come from studies of low-performing and special education students, and because general-education students do seem to succeed with the less user-friendly instruction, many educators have concluded that user-friendly instruction is only appropriate for those children of lower ability and it is inappropriate for higher-performing students.

However, the observation that high-performing students succeed in less user-friendly instruction may result from the presentation of tasks that are easy for those students relative to their ability. Perhaps providing more user-friendly instruction for high-performing students on content that is more complex relative to their ability is superior to less user-friendly methods, just as it is for low-performing students. If this hypothesis is true then the quest for world-class standards is better guided by using user-friendly instruction to teach all content that is complex relative to the ability of the student.

The Systems Impact videodisc materials used in the DI treatment of the present study have been found effective not only with older at-risk and special education students but also with younger gifted students (Carnine, 1992). The interaction of the programs' effectiveness when younger learners representing both these populations were taught in the same classroom environment was tested in the first year of this study.

A group of 60 sixth-graders were randomly assigned to the DI and NCTM treatments. Each class contained 5 special education students and 2 talented and gifted students. (A third teacher in his first year of teaching, was not included in this experiment. He received a homogeneous group of average sixth graders leaving more of the extremes of the population to the two experienced teachers in the study.) A pretest was administered to both groups (the Level 5 Inventory from the Scott Foresman program which sampled a wide-range of mathematical skills). Based on the pretest, students in each treatment group were identified as being in the lower-performing half of the class or the higher-performing half of the class. The DI group learned from the Systems Impact *Mastering Fractions, Mastering Decimals and Percents*, and *Mastering Ratios and Word Problems* videodisc programs. The NCTM students engaged in a large number of small group hands-on activities using manipulatives and were assigned mathematics projects for homework. The instruction for each group was consistent with that described later in the methods section (see the checklists for each treatment).

After nearly one year of instruction, the performance of the sixth-grade DI group was significantly higher than that of the group learning from the NCTM program in each comparison. Comparisons were further analyzed by low- and high-performing subgroup. Not only was the DI program more effective for the high performers, as well as the low performers, but the low performers also scored significantly higher than the high-performing half of the NCTM group on three of four measures. The results on four measures are graphically displayed below: performance on the NCTM program mea-

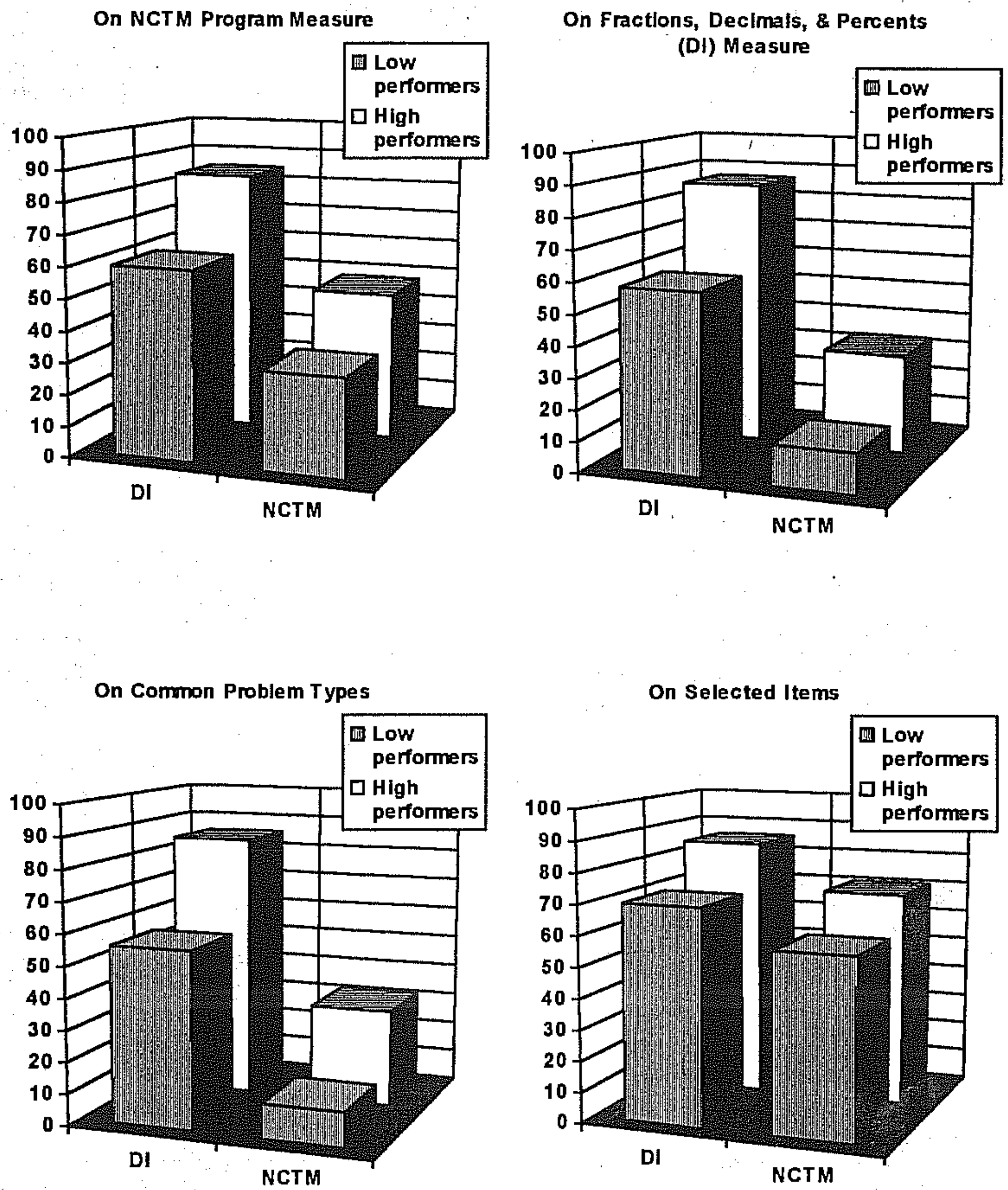


Figure 1. Evaluation of Sixth Grade During First Year of Implementation

sure; performance on the fractions, decimals, and percents measure, which aligned most closely with the considerate DI instruction; performance on problem types that were taught in both treatments (common problem types); and performance on items that were selected by the NCTM teacher as problem types on which the students receiving the NCTM treatment would be expected to do well (selected items).

When compared with the performance of the previous sixth grade in the same school on a year-end measure given to both groups, the DI treatment resulted in a significant reduction in the number of low-performing and "at-risk" students. Only 5% of the DI group scored in the range of the low-performing half of the previous 6th grade on the same test. On the other hand, the performance of the NCTM group was comparable to that of the previous sixth grade. Mainstreamed special education students worked remarkably well with the rest of the class in the user-friendly DI treatment. The resource room teacher reported that the frequent interaction provided by the DI program helped improve the behavior and work habits of students who previously had difficulty attending in class.

METHOD

Subjects

The entire fifth grade (N=58) of an elementary school in a Rocky Mountain city (population 100,000) participated in the study over a period of one and a half years. The socio-economic level of the school population was quite high; only 9% of the student population was eligible for free lunch. Students

Table 2. A Comparison of the NCTM Teaching Practices and User-Friendly Direct Instruction

NCTM Teaching Practices	User-friendly Direct Instruction (DI)
Not a 'tell-test' traditional lecture method, but a 'test-guide' method.	Not a 'tell-test' traditional lecture method, but a 'tell-**guide**-test' method, with **guide** being the major component.
Knowledge cannot be transferred, it must be invented by the learner. Knowledge grows out of solving problems in meaningful, natural contexts.	Knowledge is transferable. It is what one generation passes to the next. Knowledge is most efficiently transferred by systematic instruction.
Meaningful learning is accomplished by allowing students to develop, invent, construct their own unique strategies.	Meaning is taught by presenting carefully sequenced applications that require constant mindful discrimination.
Sequencing is a spiral design. Natural learning contexts are used in initial learning.	Sequencing is a mastery design. Learning contexts are controlled so they move from structured and contrived to naturalistic.
Child-directed	**Teacher-directed**
Students work in groups, engage in discussion, make presentations, take charge of their own learning.	Teacher (or curriculum) plans and structures learning for its most efficient and reliable conveyance.
Teacher gives explanations rarely. The teacher rather plans naturalistic activities that inspire a learning need.	Teacher plans and sequences explanations and applications to develop and fulfill learning needs.
Teacher presents the problem, allows and encourages students to form their own algorithms and strategies. The more algorithms and strategies they can devise, the better.	Teacher presents the algorithm or strategy, followed by a wide range of applications requiring constant mindful discrimination and integration with other strategies.
Independent problem solving is the *source* of learning.	Independent problem solving is the *application* of learning.

were randomly assigned to the two treatment groups, constructivist (NCTM) and user-friendly Direct Instruction at the beginning of fifth grade. One totally blind student was assigned to the DI treatment as a case study. His data were not included in the statistical analysis of the results, because he received uncontrolled prompting from his aide as the test items were presented to him orally. Over the period of the study, each subject received instruction in the respective treatments from two different teachers (fifth- and sixth-grade teachers).

The* Standards *are upfront about being "a new research agenda" rather than a description of the state-of-the-art in mathematics instructional research, as many assume them to be.

Treatment Conditions

The user-friendly (DI) treatment was experimentally compared with an NCTM mathematics program (Scott Foresman, 1991). Table 2 describes the experimental comparison between "user-friendly" DI and the NCTM "constructivist" practices.

User-friendly Direct Instruction. The curricular materials used in the DI treatment were primarily the Systems Impact videodisc programs. These programs begin by assuming a mastery of whole number operations. In fifth grade, the DI teacher used two videodisc programs in the mathematics series that teach fractions, and decimals and percents (*Mastering Fractions, Mastering Decimals and Percents*). The *Mastering Equations, Mastering Informal Geometry,* and *Mastering Ratios and Word Problems* programs were implemented in the second year, in grade 6.

Because the videodisc programs are modular in The *Mastering Equations, Mastering Informal Geometry,* and *Mastering Ratios and Word Problems* programs were implemented in the second year, in grade 6. Because the videodisc programs are modular in design, they do not represent a complete by application activities in interludes between use of the videodisc programs. These activities served to maintain earlier taught skills and provide new contexts for use of the knowledge the students had gained. These culminating activities were somewhat similar to the activities used in the NCTM instruction as the main vehicle for learning. Below is the implementation checklist that was used to check treatment fidelity.

DIRECT INSTRUCTION IMPLEMENTATION CHECKLIST

Observer:__________________________

Teacher: __________________________

Date: ____________

A. Independent workcheck (√ if done by teacher)

______ 1. Points or credit is given for accuracy on worksheets, either at end of lesson or beginning of next lesson.

______ 2. Workcheck is efficient (about 10 min.—no lengthy explanations from the teacher).

B. Lesson presentation

______ 1. Teacher moves among students, observing their work.

______ 2. Oral responses are required.

______ 3. Students correct all mistakes immediately.

______ 4. Students are referred to the videodisc for instructions and demonstrations.

______ 5. Extra explanations from the teacher are infrequent, brief, and use wording consistent with the video disc presentation.

______ 6. Only a reasonable amount of time is allowed for students to work problems. (Teacher does not necessarily wait for slowest person to finish.)

______ 7. Teacher tells students how they are doing.

______ 8. Teacher checks student performance on all Check Student Performance screens and uses appropriate remedies.

C. Independent work.

______ 1. Teacher provides extra help for low performers during seatwork using wording consistent with the videodisc presentation.

D. General.

______ 1. The videodisk program is used at least 4 times a week.

______ 2. Special remediation sessions are provided for students who consistently score below criterion.

NCTM instruction. The teachers responsible for the NCTM instruction were well-trained in the philosophy and ideals of constructivism. As a resource, the teachers used the newly adopted Scott Foresman 1991 basal mathematics program, which was designed to apply the NCTM *Curriculum and Evaluation Standards* (1989). The Scott Foresman program did not use a mastery design, but rather "spiraled" frequently through topics. Each day's lesson seemed quite different from the previous day's lesson, providing plentiful opportunities for inventive thinking, which the teacher highly praised. The teacher assigned daily problem-solving activities for students to do with their parents as homework. Students were seated in groups of four. Most lessons involved both whole group and small group discussion. The teachers maintained firm control and classes were characterized by relevant, on-task behavior.

CONSTRUCTIVIST (NCTM) TEACHING PRACTICES IMPLEMENTATION CHECKLIST

Observer: ______________________

Teacher: ______________________

A. Classroom tasks...(√ if done by teacher)

_______ 1. ...are characterized by *variety* in materials (problem booklets, computer software, practice sheets, puzzles, manipulatives, calculators, textbooks, teacher-made)

_______ 2. ...*often* allow more than one legitimate way to approach the problem, or more than one solution to the problem.[a]

_______ 3. ...foster problem solving and reasoning in a real life,meaningful context *whenever possible*.[b]

_______ 4. ...*sometimes* grow out of students' conjectures or questions and interests.[c]

_______ 5. ...include discussion, either in small or whole group format.

B. The teacher...

_______ 6. ...encourages students to talk, model, and explain, rather than teacher doing it all.[d]

_______ 7. ... leads and keeps student discourse focused by...

_______ a. ...picking up on some student ideas, leaving others behind.

_______ b. ...asking clarifying or provocative questions.[e]

_______ c. ...providing information.

_______ d. ...letting students struggle.

_______ 8. ...provokes students reasoning by asking questions like "why""explain that," "how did you reach that conclusion"?

_______ 9. ...gets each student to participate.

_______ 10. ...listens carefully to students' ideas.

11. ...requires students to respond in a variety of forms (orally, concrete or representational, written or pictorial)

[a] For example, rather than require students to find the mean, median, and mode for a set of data (a task requiring student to produce right answers), the task would require them to decide whether to calculate means, medians, or modes for given sets of data and particular claims to be made about the data, to calculate those statistics, and then defend their decisions.

[b] For example, rather than simply practice addition and subtraction facts to develop automaticity, the students would roll dice as part of an investigation of probability—a task that requires rapid fire addition.

[c] The teacher helps students learn to conjecture, invent, and solve problems by asking questions such as "What would happen if...?" "What are some possibilities here?" "How did you think about that problem?" "What is alike and what is different about your solution method and another student's method?"

[d] The teacher develops a sense of community in working out mathematics problems by asking questions such as "What do others think about what Janie said?" "Do you agree? Disagree?" "Does anyone have the same answer but a different way to explain it?" "Can you convince the rest of us that that makes sense?"

[e] The teacher helps students to connect mathematics, its ideas, and its applications by asking questions such as "How does this relate to...?" "What ideas that we have learned before were useful in solving this problem?" "Have we ever solved a problem like this one before?" "Can you give me an example of...?"

Procedures

The two 5th grade teachers had taught different grade levels the previous year. Comparisons of the mathematics performance of their previous students with that of students taught by other experienced teachers indicated no significant differences.

One 6th grade teacher was particularly knowledgeable of the NCTM methods and had piloted the NCTM program the previous year. She was responsible for training many teachers in the district in the new method that was now adopted district-wide. She also strongly preferred to use the district's newly-adopted mathematics basal and implemented it with zeal, spending long hours after school preparing lessons. In contrast the DI teacher could be described as competent but dispassionate.

The 6th grade DI and NCTM teachers were also equivalent in mathematics teaching skill based on the achievement results of their previous class of students. The DI teacher's previous class had a mean total mathematics percentile of 61 on the Iowa Test of Basic Skills (ITBS) with 32% of her students scoring below the fiftieth percentile. The NCTM program teacher's previous class had a mean of 63 with 33% of her students scoring below the fiftieth percentile.

The DI and NCTM teachers were observed twice a week initially, until all the guidelines for implementing the videodisc programs were followed. Thereafter, the teachers were observed monthly.

Measures

The Inventory from the 4th-grade level of the Scott Foresman text was used as a pretest (internal consistency reliability was .83, calculated using Pearson's coefficient alpha). The 4th-grade level was selected because it sampled a very wide range of mathematical skills and was at a level that would result in wider variation in the students scores. The pretest was administered prior to implementation of the differential treatments.

Two mid-term tests were administered in May of the first year of instruction. Posttests were administered in February of the second instructional year, with the exception of the Iowa Test of Basic Skills, which was administered in April of the second year.

Mid-term DI criterion-referenced test. The experimenters developed a 29-item test covering the skills that were taught in the DI treatment. The items tested students' ability to manipulate quantities involving fractions, decimals, and percents.

Mid-term NCTM criterion-referenced test. The 22-item year-end cumulative test from the Scott Foresman text was used as a criterion-referenced measure of the material covered in the NCTM program.

NCTM problem-solving posttest. The experimenters designed a 40-item measure that sampled the items described in the NCTM standards. These problems were non-routine or atypical; however, they were closely aligned with the NCTM curriculum. For example, one problem presented a visual display of a cube inside another cube with sides twice the size of the sides of the small cube, and the question was: If you double the sides of this square, how many times bigger is the surface area of the new square, and how many times bigger is the volume of the new square? Another problem involved categorizing a set of fractions according to their most approximate size (0, 1/2, 1, 1 1/2). These problems were taken directly from the NCTM standards for grade six.

Algebraic word-problems posttest. The experimenters designed a 27-item algebraic word problem test that aligned with the DI treatment. The problems were routine or typical of the kinds of problems one encounters in school and real life. For example, "Kyle paid 4/5 of the total bill. The total bill was $10.25. How much did Kyle pay?" or "Three times a number is 4 less than 25. What is the number?"

Woodcock Johnson applications scale, posttest. The applications subscale of the Woodcock Johnson was used as a standardized assessment of the breadth of problem solving ability. Problems on the Woodcock Johnson sample a wide range of problem solving skills. However, the test was administered to the groups as a whole, rather than individually, as the standardized procedures would require.

Iowa Test of Basic Skills, Level 14, Form G (ITBS). The Level 14 Form of the ITBS is usually administered to the eighth grade. We chose to administer this level of the test to the sixth grade because we believed the test would be more sensitive to the more advanced mathematics skills that we believed the groups were learning.

The ITBS consisted of three subscales: Concepts, problem solving, and computation. The concepts subscale covered primarily concepts in measurement and geometry. The testing manual comments that students' ability to manipulate quantities is no longer tested in the meaningful context of the problem-solving subscale. Instead, what was once measured in the "problem-solving" subscale has now been separated into two subscales, problem solving and computation:

> "In earlier forms of the tests, computation skills had been tested in a functional setting in the problem-solving tests. Although the problem-solving tests in Form G... still require computation, the fundamental operations and concepts involved are those which, in most cases, have been introduced at least a year prior to the grade for which a level of the test is primarily intended." (ITBS, 1986, p. 71)

The manual makes this comment about the computations subscale:

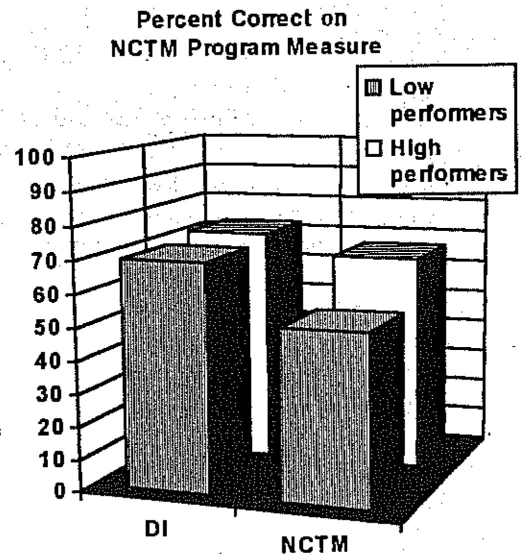
> "In earlier editions of the ITBS, there was no separate comptation test. Instead, computation was measured in a meaningful setting in [the problem-solving subscale].... This is still true to some extent in the current forms in that pupils have to be able to compute in order to get the correct answer. However, in order to obtain a purer measure of problem solving, the computational level required in the problem-solving test has been reduced significantly. The construction of a problem-solving test relatively independent of computation skills necessitated the addition of a separate computation test." (ITBS, 1986, p. 72)

RESULTS

Fifty-eight subjects completed year 1 and were assessed on the mid-term measures. Forty-five treatment subjects remained at the end of the second year, 19 in the DI treatment and 26 in the NCTM treatment. However, class sizes for each treatment were larger; there were now 33 students assigned to each class. The new students generally came from other classes and schools where the NCTM approach was being implemented. This created a special problem for the DI teacher. She had to continue providing the 19 original DI subjects with additional levels of the DI treatment; along with 14 additional students who had not mastered the preskills.

Is User-Friendly DI Instruction Only for Low Performing Students?

Because the number of subjects in each treatment was not adequate at the end of the second year for a separate analysis of the low- and high-performing groups, we analyzed only the midterm data by these subgroups. In May the DI and NCTM midterm criterion-referenced tests were administered. The graph below displays the performance of the ability subgroups on the DI and NCTM program tests. Differences for low performers were significant on both measures, favoring the DI group. Differences for high performers were significant only on the DI measure, again favoring the DI group. However, the low-performing half of the DI group also scored significantly better on the DI measure of fractions, decimals, and percents than the high-performing half of the NCTM group.



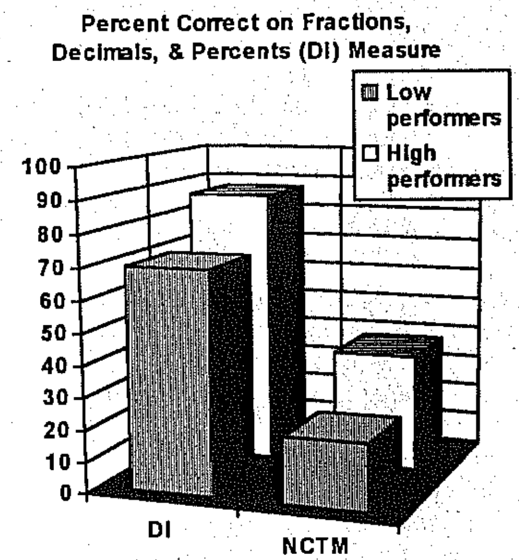


Figure 2. Performance at the End of Fifth Grade

Do the NCTM Standards Result in Better Problem Solving?

We administered the problem-solving posttests near the end of the second year. An MANOVA indicated that overall the DI group scored significantly higher than the NCTM group (Wilks lambda = .56, $\underline{F}(6,33) = 4.35$, $\underline{p} < .01$). Table 3 displays the univariate F-test results for each of the separate tests. The DI subjects scored significantly higher on the algebraic word problems test and on the computation subscale of the ITBS. Figures 3, 4, and 5 graphically display these results.

User-friendly Direct Instruction resulted in better overall problem-solving performance than the NCTM teaching practices.

Table 3. Mean Score and Standard Deviation of the NCTM and DI Groups on All Measures

	NCTM Mean (SD)	DI Mean (SD)	F(1,38)	p
Pretest	28.0 (4.5)	27.7 (5.8)	.014	.91
NCTM Problems (nonroutine)	27.7 (7.6)	26.8 (7.3)	.13	.72
Algebraic Word Problems (routine)	12.6 (5.5)	17.7 (5.4)	9.96	.003*
Woodcock Johnson Applications	10.1 (4.0)	11.8 (3.4)	2.06	.16
ITBS Concepts	22.9 (9.0)	26.2 (6.8)	2.41	.13
ITBS Problem Solving	16.4 (7.0)	17.1 (5.4)	.28	.60
ITBS Operations	21.2 (8.3)	28.8 (9.1)	8.9	.005*

* significant

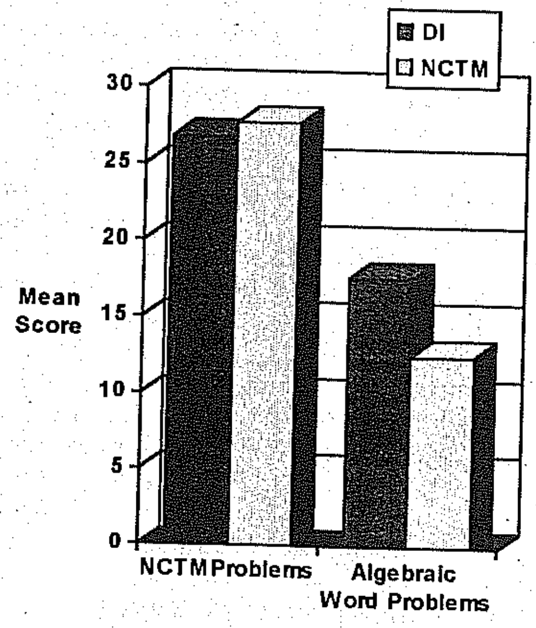


Figure 3. Program Aligned Problem Solving Measures.

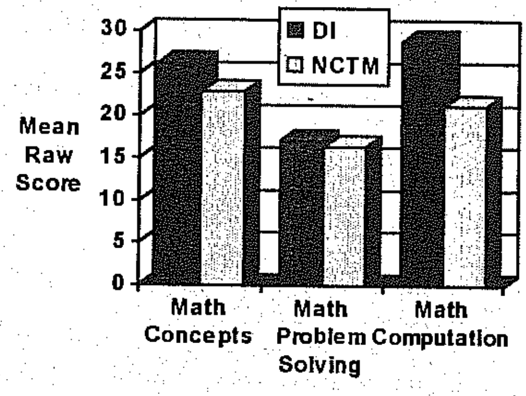


Figure 5. Eighth-Grade ITBS Achievement Test

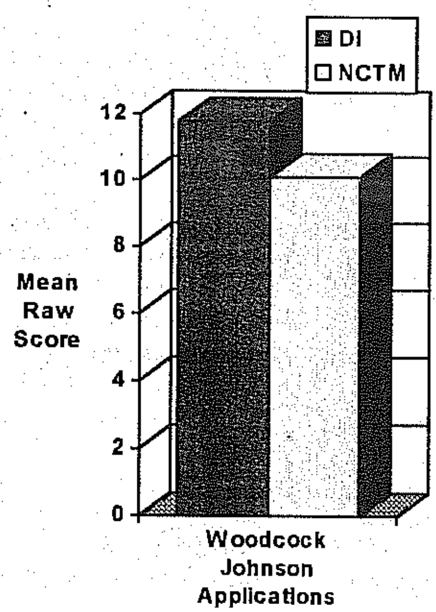


Figure 4. Breadth of Problem Solving

DISCUSSION

An uncontrolled variable was the amount of homework assigned in the 6th grade classes. The teacher and the parents of the NCTM 6th grade group reported that these subjects did approximately 3 hours of mathematics homework each week. In contrast, the DI classes were assigned no homework. Also the NCTM 6th grade teacher emphasized the integration of mathematics in other subjects and strongly reinforced mathematics when studying science, social studies, and so on. The DI teacher did not integrate in this way. The overall time students spent engaged in mathematics activities seemed much higher for the NCTM treatment.

In spite of these biases favoring the NCTM treatment, the results of this study do not support the exclusive use of the NCTM teaching practice recommendations as the most effective means for achieving the NCTM problem-solving goals. User-friendly Direct Instruction resulted in significantly better overall problem solving performance than the NCTM teaching practices. Furthermore, not only low-performing, but also high-performing students achieved greater gains in mathematics skills with user-friendly Direct Instruction than with the NCTM teaching practices, according to the midterm criterion-referenced tests.

Though the DI students spent little to no time engaged in nonroutine problems (problems that do not lend themselves to an algorithmic solution), the scores for the DI students were not significantly different from those for the NCTM group, who spent a great deal of time engaged in nonroutine problems on the NCTM measure. For the DI treatment, the problems on the NCTM measure were completely new and unpracticed. The NCTM treatment had, however, practiced problems in these forms. Therefore, the NCTM test required the DI group to make further transfer of their learning than it required of the NCTM group. The fact that performance did not differ significantly in this area may indicate that students do not really develop a deep understanding of mathematics by working nonroutine types of problems. The NCTM recommendation emphasizing nonroutine problems in both mathematics instruction and assessment should be carefully reconsidered. In addition to their possible instructional benefit, routine problems also seem more representative of the types of mathematics problems students will encounter in real life. Learning common patterns of mathematical problems may facilitate, rather than hinder creative problem solving and the students' ability to solve nonroutine problems. This question warrants further research.

Results for high-performing students were favorable. On five of the six measures comparing the performance of low- and high-performing groups, the DI high performers scored significantly better than the NCTM high performers. On three of the six measures, the low-performing half of the DI group scored significantly higher than the high-performing half of the NCTM group.

Results were also favorable for special education students. One student who had an IEP in mathematics achieved an above-grade-level score in mathematics after receiving instruction from the DI programs. A blind student who worked with an aide in the DI treatment showed great improvement in mathematics achievement and in his attitudes towards mathematics. He seemed to get a boost from responding with the class to the frequent questions presented by the DI programs. His aide commented that the auditory stimulus from the videodisc program seemed so much clearer than the normal verbal instruction that he hears in the classroom, regardless of the fact that he could not see the video presentation.

The *Mastering Fractions* program seemed better placed in fifth grade than it was in sixth. The first group of sixth graders who learned from the *Mastering Fractions* program had already been exposed to fractions. For this reason, some sixth-grade students in the first group complained that they already knew fractions, though their pretest scores indicated their knowledge was spotty. However, their prior knowledge of fractions was much more variable than their prior knowledge of other concepts taught in the videodisc programs, for example, how to set up ratios and equations. In these other programs where students had no prior knowledge, their response was very positive. The *Mastering Fractions* program was better placed in fifth grade because there none of the students claimed to know much about fractions and were eager to learn from the videodisc. The groups were heterogeneous in this study. We believe better results would be achieved by grouping students according to their specific skill needs. For example, those students who have mastered fractions should be able to move beyond fractions to learn higher mathematics where fractions are used in a sophisticated way, such as setting up ratio or algebraic equations.

Note: Systems Impact videodisc programs are available from:
BFA Educational Media
2349 Chaffee Drive
Saint Louis, MO 63146
1-800-221-1274

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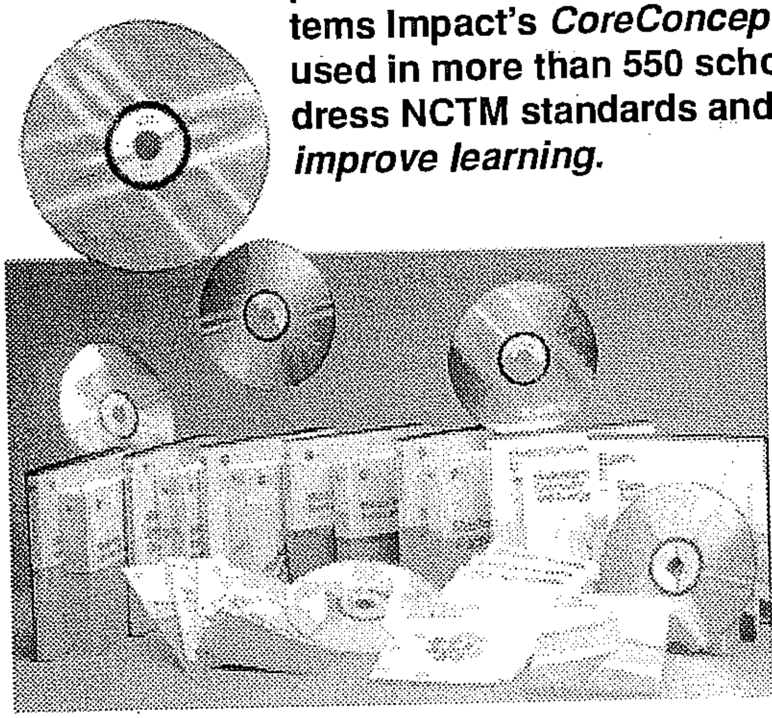

Every lesson in the *CoreConcepts*™ Math and Science series fits into a 35-45 minute period and each program includes an **Instructor's Manual** and 30 **Student Response Booklets.**

Each program operates on the Pioneer® LD–V2200 and other consumers and industrial laser videodisc players, and is designed for use with the wireless remote control or the Pioneer ® UC–V104 Laser BarCode Scanner. Disc players and scanners are available for purchase through BFA.

Programs may be purchased individually or in sets.

Educators rave about *CoreConcepts*™ user friendly programs:

"CoreConcepts™ allow teachers to spend their time monitoring and interacting with their students. They remain in full control of the learning process."
–Supervisor of Externally Funded Projects,
Compton School District,
California

"As a staff development tool, the discs model good teaching and learning strategies. By using the Systems Impact Programs, effective teachers are becoming even more effective."
–Principal,
Cobb County Schools,
Georgia

"Remedial and Chapter 1 students in the Mastering Fractions program jumped more than 300% in fraction skills and improved 43% in the state math tests (TELS)."
–Bethel Park Schools District,
Pennsylvania

INTRODUCTION

This video tape training series was produced by authors / implementors of the Direct Instruction Model on a not-for-profit basis so that high quality staff development might be widely disseminated. The first five tapes in the series were designed as intensive pre-service training on READING MASTERY I/FAST CYCLE I for personnel—teachers, instructional assistants, tutors, supervisors, administrators and other implementors—who may have had no prior knowledge or experience with any Direct Instruction program. The tapes may also be used as a thorough review of implementation procedures with personnel who have had experience with Direct Instruction. Another way the series may be used is to provide a model for consultants and trainers. Obviously, the optimum way to use the pre-service tapes with instructional personnel is in a group before instruction begins with children. The five pre-service tapes run slightly more than 6 hours; however, since the off-screen practice sessions are an essential part of the training, the total pre-service time required is at least 11 hours. Group sessions are highly recommended, because each person must have a partner or small group for practice sessions. When new personnel enter an on-going implementation during the school year, it's helpful if arrangements can be made for a peer partner or mentor.

[The first tape which runs 37 minutes may be used separately as an orientation to Direct Instruction for school personnel, parents and community members or as a vehicle for public relations. Beginning with Tape 6, the series continues as in-service training. Sessions should be planned for an hour or more, scheduled weekly during the first months of school, and then as often as necessary to "stay ahead of the children." Training segments cover approximately a 10-lesson span. The whole series may be used as the basis of a college credit course.]

Scope and Sequence
READING MASTERY I /FAST CYCLE I
PRE-SERVICE TRAINING VIDEOS

Tape 1* (37 min.)

Introduction to Direct Instruction

Development History, Research—Data, Philosphy and Goals, Design Strategies
Message from Siegfried Engelman

READING MASTERY I-VI Overview

Tape 2* (69 min. + Practice Sessions)

Materials Orientation—Level I

Prerequisite Teacher Skills

Sounds—practice
Blending—practice
Reading words spelled phonetically—practice

Prequisite Organization/Management Strategies

Placement testing—practice
Grouping students
Scheduling

Tape 3* (1:56 + Practice Sessions)

Training and Practice segments—Level I Pre-reading Exercises

Pronunciation
Say the Sounds
Say It Fast
{moved to next tape during revision—Say the Sounds-Say It Fast}

Tape 4* (1:39 + Practice Sessions)

Continuation of Training and Practice

Say the Sounds-Say It Fast}
Symbol Identification
Sounds Firm Up
Sequencing Games
Workbook Applications
- Sounds
- Writing

Tape 5* (45 minutes)

Classroom Management and Organization

Demonstration with children

Lesson 12 READING MASTERY I=lesson 2 FAST CYCLE I

In-program mastery testing

*** *total actual running time—6 hours; with practice, 5 tapes = 11+ hours***
tapes 1-5 cover pre-reading skill lessons. Series is continued on tape 6—in-service training.

ADI MATERIALS PRICE LIST

***The Surefire Way to Better Spelling** (1993)*
by Robert C. Dixon
Membership Price: $8.75 — *List Price: $10.75* __________

***Theory of Instruction** (1991)*
by Siegfried Engelmann & Douglas Carnine
Membership Price: $32.00 — *List Price: $40.00* __________

***Direct Instruction Reading** (Revised, 1990)*
by Douglas Carnine, Jerry Silbert, & Ed Kameenui
Membership Price: $32.00

Order from:	MacMillan Publishing 1-800-257-5755 DI Reading, ISBN 0-675-21014-3

***Direct Instruction Mathematics** (Revised, 1990)*
by Jerry Silbert, Douglas Carnine, & Marcy Stein
Membership Price: $32.00

Order from:	MacMillan Publishing 1-800-257-5755 DI Math, ISBN 0-675-21208-1

***Teach Your Child to R[illegible] 00 Easy Lessons** (1983)* SOLD OUT
by Siegfried Eng[illegible] [illegible]us Haddox, & Elaine Bruner
Membership [illegible].95 — *List Price: $17.95* __________

***Structuring Classrooms [illegible]mic Success** (1983)* SOLD OUT
by Stan Paine, J. R[illegible] [illegible]osellini, L. Deutchman, & C. Darch
Membership Pric[illegible] — *List Price: $14.00* __________

***War Against the Schools' Academic Child Abuse** (1992)*
by Siegfried Engelmann
Membership Price: $14.95 — *List Price: $17.95* __________

***Becoming a Nation of Readers** (1985)*
The Report of the Commission on Reading
Membership Price: $4.00

Mail orders to: $4.50	University of Illinois—BNR PO Box 2276, Station A Champaign, IL 61825-2276

***Beginning to Read: Thinking and Learning About Print** (1990)*
by Marilyn Jager Adams (A summary by the Center on Reading)
Membership Price: $8.00

Mail orders to: $5.00	University of Illinois Summary PO Box 2276, Station A Champaign, IL 61825-2276

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